完全流体のポテンシャル流を用いたCFRPの電流解析手法

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1.緒　言

2.直交異方性場の電流

3.ポテンシャル流れを用いた電流解析

4.有限要素法との比較

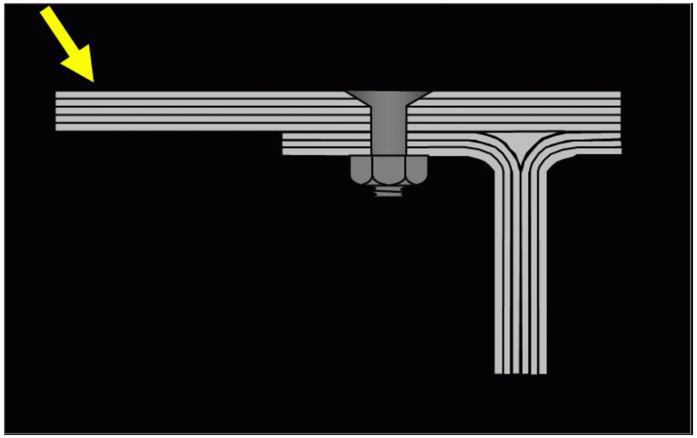
5. 厚板CFRPの積層理論

6. 結　論

Design Informatics (DI) Lecture Series 1 @ Kitami IT

1. 緒　言

Lighting strike

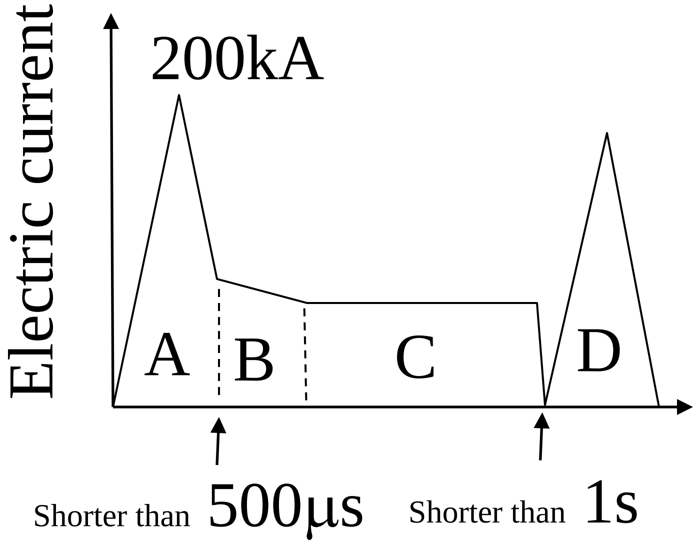


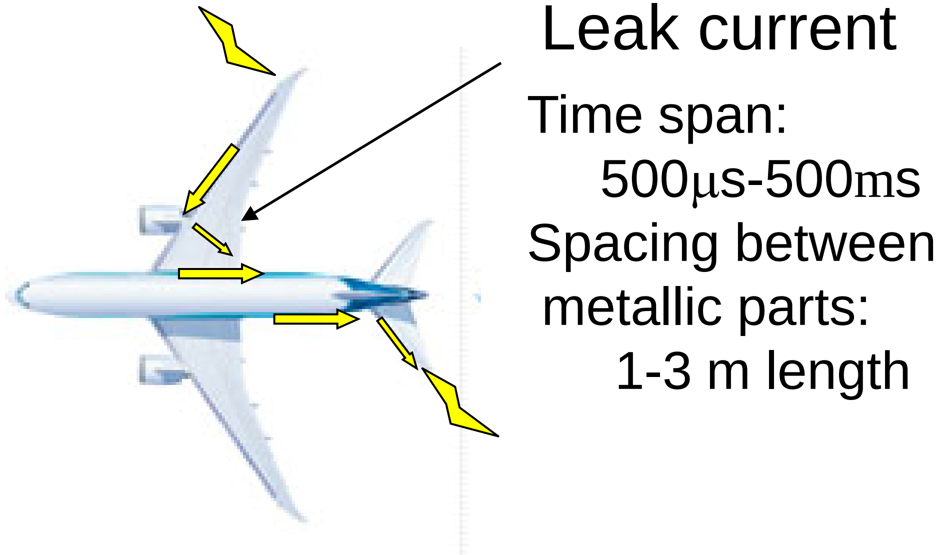
Thermal Spark

3-D FEM analysis of the real composite wing is impossible

Electric current analysis (1-2 m length, t=10 – 20 mm **)**
Electric current analysis is indispensable for spark analysis.

Lightning strike



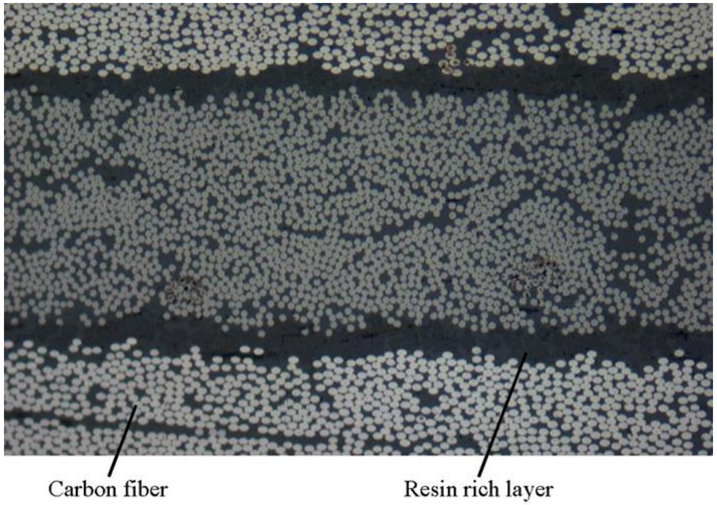


Order of frequency: 10kHz

A steady state current analysis is applicable to obtain rough estimations of the leak current except for the rising edge

Skin effect $$\delta = \sqrt{\frac{2}{\sigma\mu\omega}} \approx 60\,mm$$

Toughened CFRP laminates



T300/Epoxy $130^{\circ}C\ cure$

V_f	σ_0 (S/m)	σ_{90} / σ_0	σ_t / σ_0
0.40	3700	1.8×10^{-4}	1.6×10^{-5}
0.47	4600	1.1×10^{-3}	2.2×10^{-4}
0.62	5500	3.7×10^{-2}	3.8×10^{-3}

IM600/133 $180^{\circ}C\ cure\ (High\ toughness)$

(S/m)

$\sigma_x\ (fiber)$	$\sigma_y\ (transverse)$	$\sigma_z\ (tickness)$
36000	1.15	0.0018

Very small conductance in the thickness direction

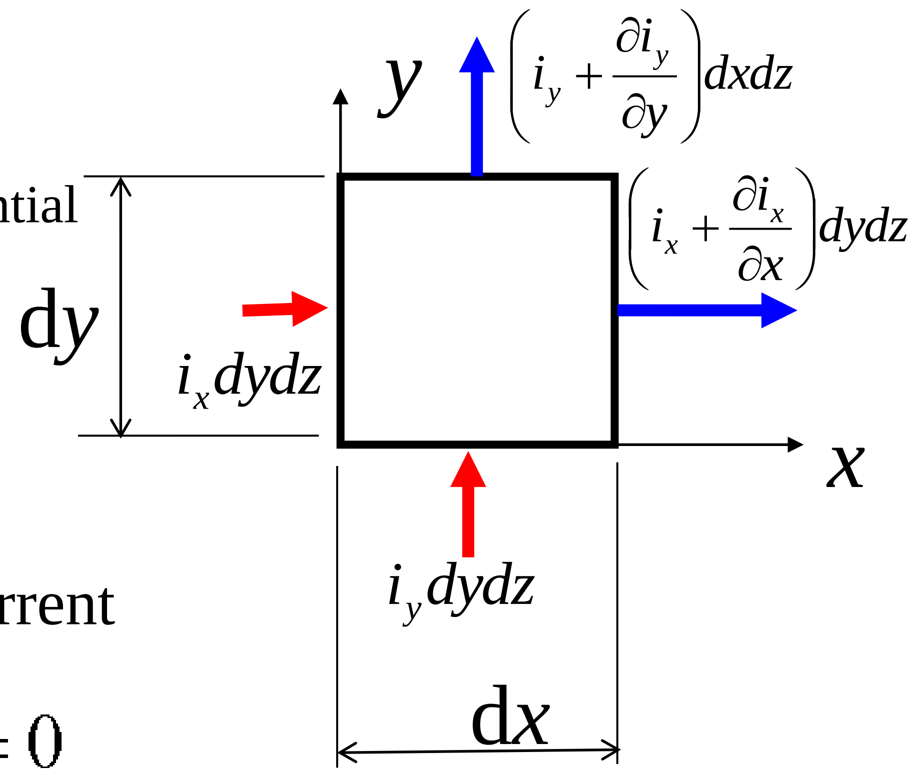
目　的

- 直交異方性場の電流解析
- ポテンシャル流れを用いた簡易解析
- FEMとの比較検討

2.直交異方性場の電流

Electric current density is calculated
using partial differentiation of electric potential

$$i_x = -\sigma_x \frac{\partial \phi}{\partial x} \quad i_y = -\sigma_y \frac{\partial \phi}{\partial y}$$



Equation of continuity of electric current

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\sigma_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\sigma_y \frac{\partial \phi}{\partial y}\right) = \sigma_x \frac{\partial^2 \phi}{\partial x^2} + \sigma_y \frac{\partial^2 \phi}{\partial y^2} = 0$$

Coordinate conversion

$$\xi = \frac{x}{\sqrt{\sigma_x}}, \quad \eta = \frac{y}{\sqrt{\sigma_y}}$$

$$\sigma_x \frac{\partial^2 \phi}{\partial x^2} + \sigma_y \frac{\partial^2 \phi}{\partial y^2} = 0$$

Orthotropic

$$\Downarrow$$

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0$$

Isotropic
(Laplace's equation)

Potential ϕ satisfy the Laplace's equation on the coordinate of ξ and η.

3.Electric current flow analysis using potential flow

Potential flow of perfect fluid without eddy current

Velocity potential

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}$$

$$i_x = -\sigma_x \frac{\partial \phi}{\partial x} \quad i_y = -\sigma_y \frac{\partial \phi}{\partial y}$$

Eqation of continuity

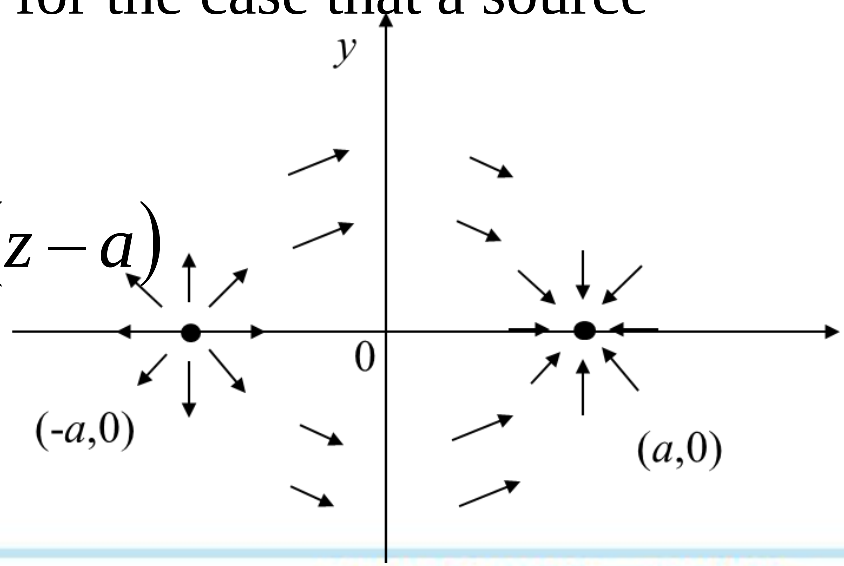
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Complex potential function is obtained for the case that a source locates (-*a*,0) and a sink locates (*a*,0).

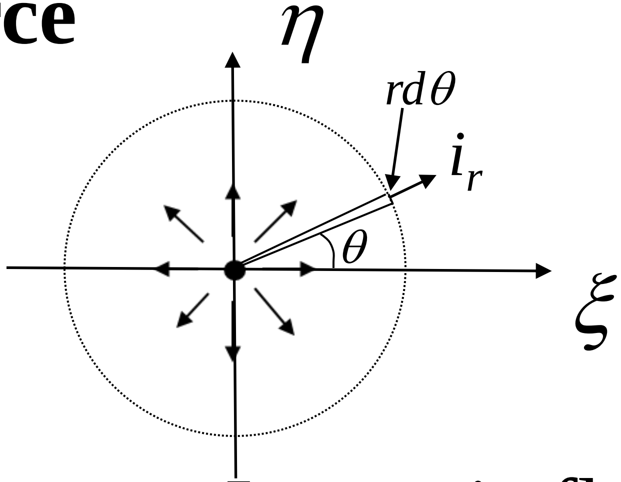
$$W(z) = \frac{q}{2\pi}\log(z+a) - \frac{q}{2\pi}\log(z-a)$$

Source

Sink



Let us consider a case of source



Isotropic flow

$$\phi = q \log r$$

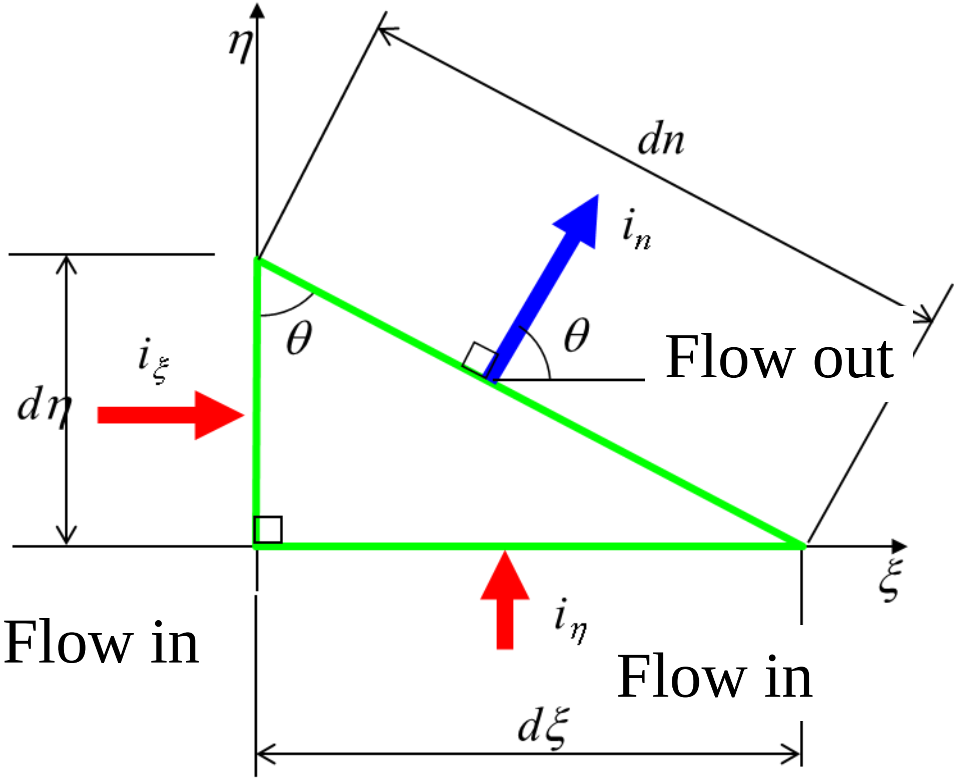
$$r = \sqrt{\xi^2 + \eta^2}$$

$$i_\xi = -\sqrt{\sigma_x \sigma_y}\frac{\partial \phi}{\partial r}\frac{\partial r}{\partial \xi} = -q\sqrt{\sigma_x \sigma_y}\frac{\xi}{r^2}$$

$$i_\eta = -\sqrt{\sigma_x \sigma_y}\frac{\partial \phi}{\partial r}\frac{\partial r}{\partial \eta} = -q\sqrt{\sigma_x \sigma_y}\frac{\eta}{r^2}$$

Integral of flow in every direction

$$I = \int_0^{2\pi} i_r r d\theta \qquad \therefore q = -\frac{I}{2\pi\sqrt{\sigma_x \sigma_y}}$$



$$i_\xi d\eta + i_\eta d\xi = i_n dn$$

$$n \to r$$

$$i_r = i_\xi \cos\theta + i_\eta \sin\theta$$

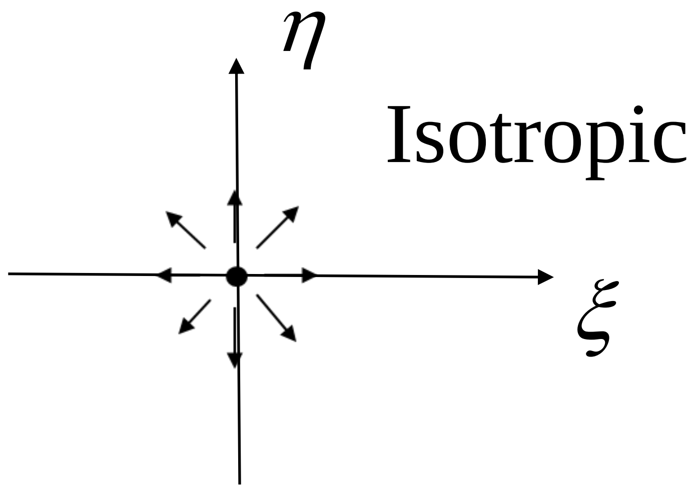
$$I = -q\sqrt{\sigma_x\sigma_y}\int_0^{2\pi}\left(\frac{\xi}{r^2}\cos\theta + \frac{\eta}{r^2}\sin\theta\right)r d\theta$$

$$= -q\sqrt{\sigma_x\sigma_y}\int_0^{2\pi}\left(\cos^2\theta + \sin^2\theta\right)d\theta$$

$$= -2\pi q\sqrt{\sigma_x\sigma_y}$$

$$\therefore q = -\frac{I}{2\pi\sqrt{\sigma_x\sigma_y}}$$

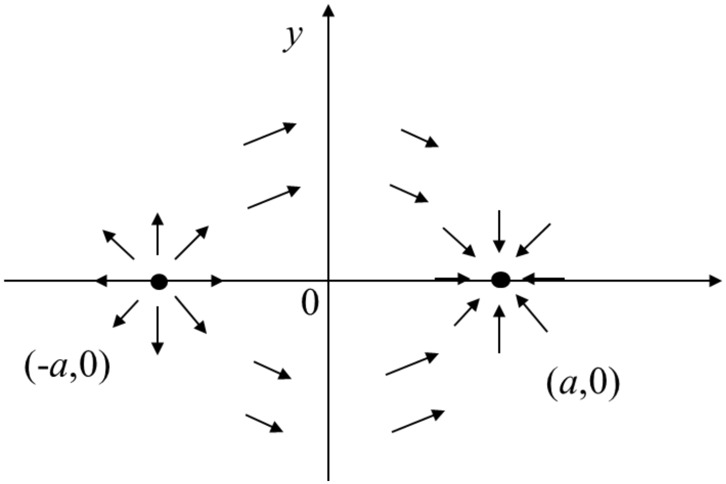
Electric current source



$$i_x = \frac{I}{2\pi\sqrt{\sigma_x \sigma_y}} \frac{x}{\dfrac{x^2}{\sigma_x} + \dfrac{y^2}{\sigma_y}}$$

$$i_y = \frac{I}{2\pi\sqrt{\sigma_x \sigma_y}} \frac{y}{\dfrac{x^2}{\sigma_x} + \dfrac{y^2}{\sigma_y}}$$

Opposite sign of the equation gives the answer of electric current sink

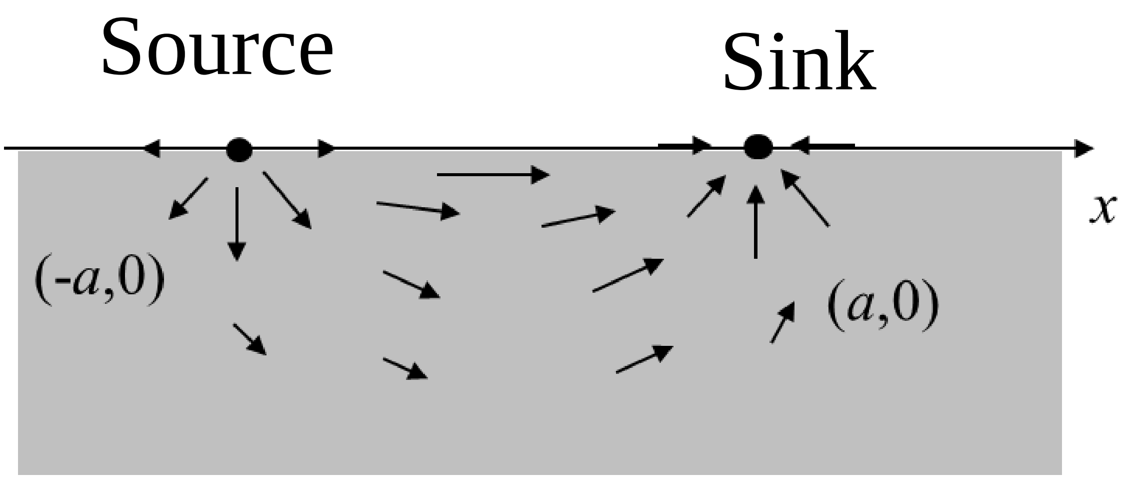


Source
+
Sink

$$i_x = \frac{I}{2\pi\sqrt{\sigma_x \sigma_y}} \left\{ \frac{x+a}{\frac{(x+a)^2}{\sigma_x} + \frac{y^2}{\sigma_y}} - \frac{x-a}{\frac{(x-a)^2}{\sigma_x} + \frac{y^2}{\sigma_y}} \right\}$$

$$i_y = \frac{I}{2\pi\sqrt{\sigma_x \sigma_y}} \left\{ \frac{y}{\frac{(x+a)^2}{\sigma_x} + \frac{y^2}{\sigma_y}} - \frac{y}{\frac{(x-a)^2}{\sigma_x} + \frac{y^2}{\sigma_y}} \right\}$$

Electric current density analysis of cross section



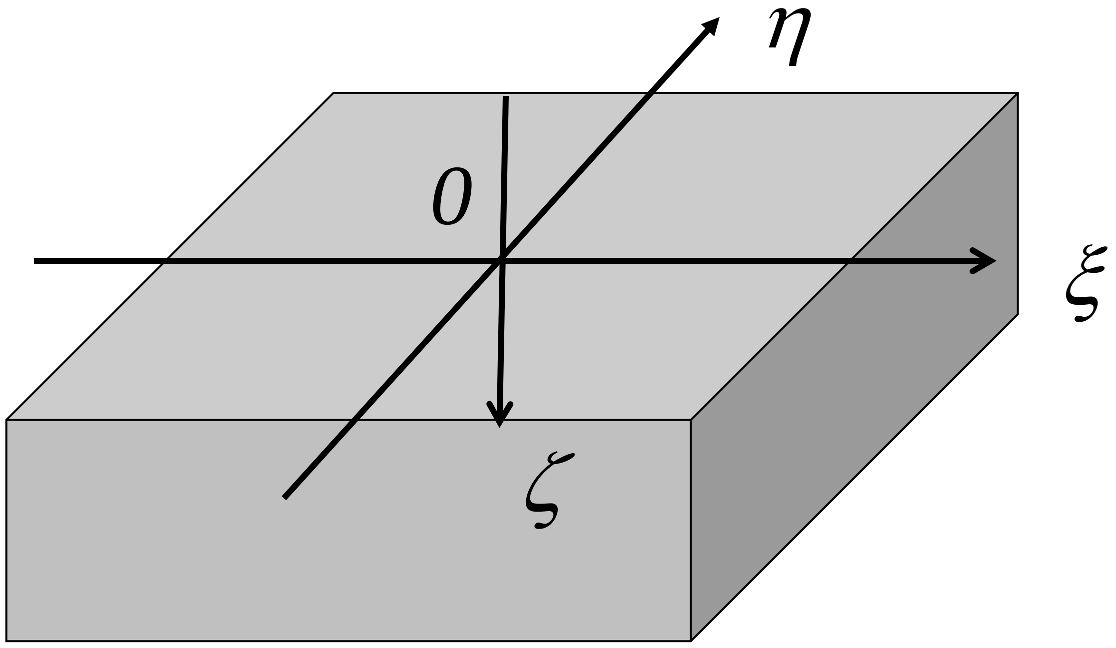
I for Half side

$I \rightarrow 2I$

$$i_x = \frac{I}{\pi\sqrt{\sigma_x\sigma_y}}\left\{\frac{x+a}{\frac{(x+a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}} - \frac{x-a}{\frac{(x-a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}}\right\}$$

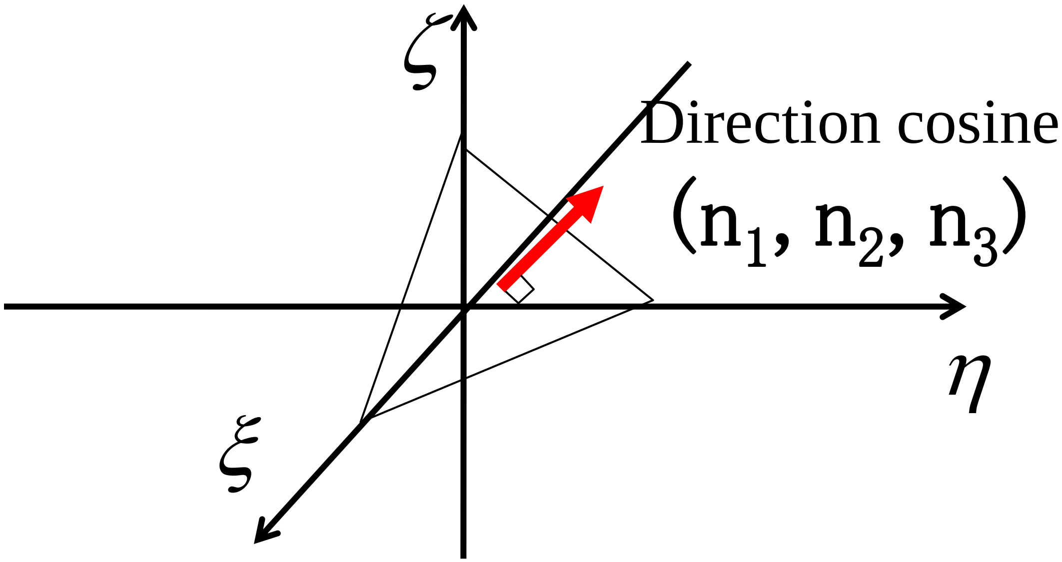
$$i_y = \frac{I}{\pi\sqrt{\sigma_x\sigma_y}}\left\{\frac{y}{\frac{(x+a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}} - \frac{y}{\frac{(x-a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}}\right\}$$

Semi-infinite body



Potential of a source for infinite body

$$\phi = -\frac{q}{r}$$



$$\xi = \frac{x}{\sqrt{\sigma_x}},\ \eta = \frac{y}{\sqrt{\sigma_y}},\ \zeta = \frac{z}{\sqrt{\sigma_z}}$$

$$i_\xi = -q\sqrt{\sigma_x \sigma_y \sigma_z}\,\frac{\xi}{r^3}$$

$$i_\eta = -q\sqrt{\sigma_x \sigma_y \sigma_z}\,\frac{\eta}{r^3}$$

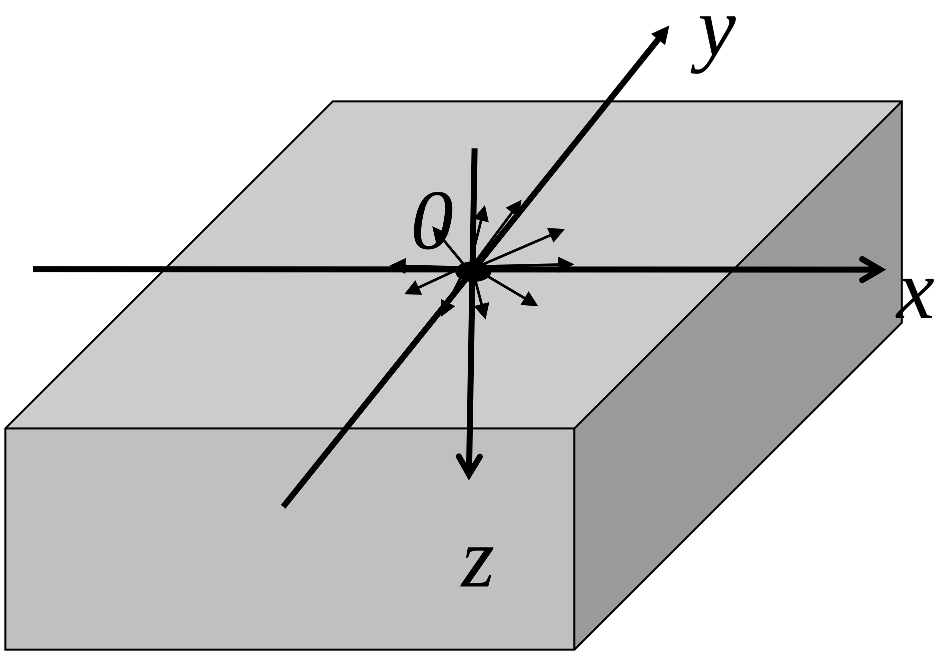
$$i_\zeta = -q\sqrt{\sigma_x \sigma_y \sigma_z}\,\frac{\zeta}{r^3}$$

$$i_r = i_\xi n_1 + i_\eta n_2 + i_\zeta n_3$$

$$\begin{aligned}
I &= \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} i_r r\cos\theta d\theta r d\varphi \\
&= \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \left(i_\xi n_1 + i_\eta n_2 + i_\zeta n_3\right) r^2 \cos\theta d\theta d\varphi \\
&= -q\sqrt{\sigma_x \sigma_y \sigma_z}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \left(n_1^2 + n_2^2 + n_3^2\right)\cos\theta d\theta d\varphi \\
&= -4\pi q\sqrt{\sigma_x \sigma_y \sigma_z}
\end{aligned}$$

Source in a semi-infinite body

$$\therefore q = -\frac{I}{4\pi\sqrt{\sigma_x \sigma_y \sigma_z}}$$



$$\begin{cases} i_x = \dfrac{I}{4\pi\sqrt{\sigma_x \sigma_y \sigma_z}} \dfrac{x}{\left(\dfrac{x^2}{\sigma_x} + \dfrac{y^2}{\sigma_y} + \dfrac{z^2}{\sigma_z}\right)^{3/2}} \\ i_y = \dfrac{I}{4\pi\sqrt{\sigma_x \sigma_y \sigma_z}} \dfrac{y}{\left(\dfrac{x^2}{\sigma_x} + \dfrac{y^2}{\sigma_y} + \dfrac{z^2}{\sigma_z}\right)^{3/2}} \\ i_z = \dfrac{I}{4\pi\sqrt{\sigma_x \sigma_y \sigma_z}} \dfrac{z}{\left(\dfrac{x^2}{\sigma_x} + \dfrac{y^2}{\sigma_y} + \dfrac{z^2}{\sigma_z}\right)^{3/2}} \end{cases}$$

$I \longrightarrow 2I$ gives an answer of semi-infinite body

Source

$+$

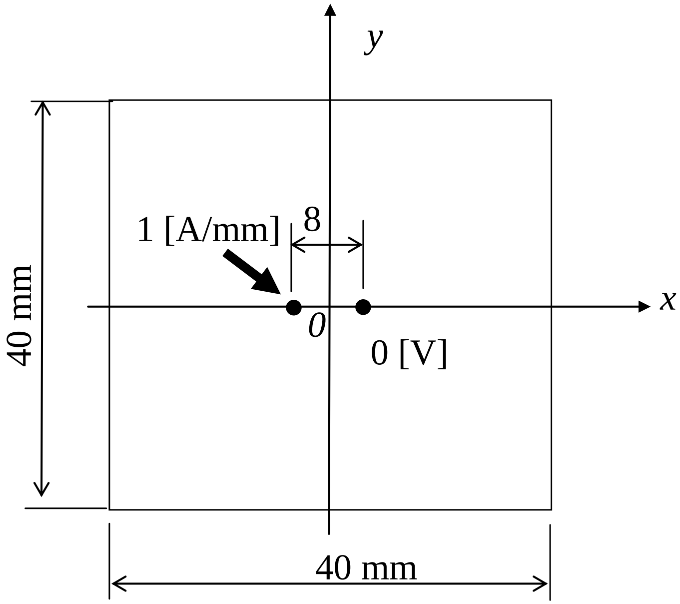
Sink

$$i_x = \frac{I}{2\pi\sqrt{\sigma_x\sigma_y\sigma_z}}\left\{\frac{(x+a)}{\left(\frac{(x+a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}+\frac{z^2}{\sigma_z}\right)^{3/2}} - \frac{(x-a)}{\left(\frac{(x-a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}+\frac{z^2}{\sigma_z}\right)^{3/2}}\right\}$$

$$i_y = \frac{I}{2\pi\sqrt{\sigma_x\sigma_y\sigma_z}}\left\{\frac{y}{\left(\frac{(x+a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}+\frac{z^2}{\sigma_z}\right)^{3/2}} - \frac{y}{\left(\frac{(x-a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}+\frac{z^2}{\sigma_z}\right)^{3/2}}\right\}$$

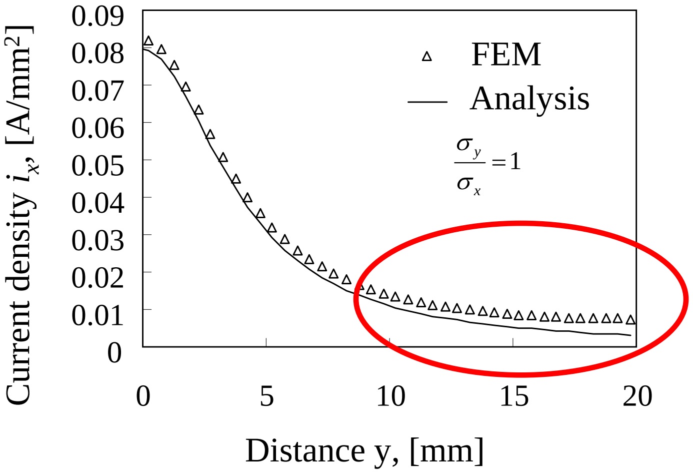
$$i_z = \frac{I}{2\pi\sqrt{\sigma_x\sigma_y\sigma_z}}\left\{\frac{z}{\left(\frac{(x+a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}+\frac{z^2}{\sigma_z}\right)^{3/2}} - \frac{z}{\left(\frac{(x-a)^2}{\sigma_x}+\frac{y^2}{\sigma_y}+\frac{z^2}{\sigma_z}\right)^{3/2}}\right\}$$

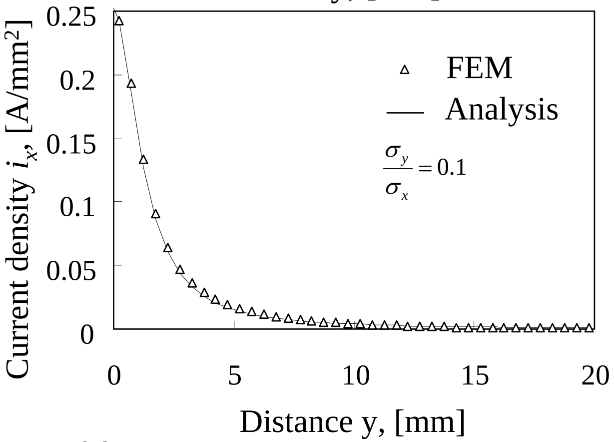
4.Comparison with FEM

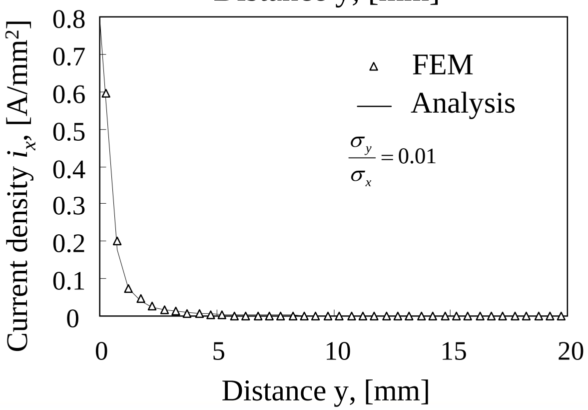


Model A

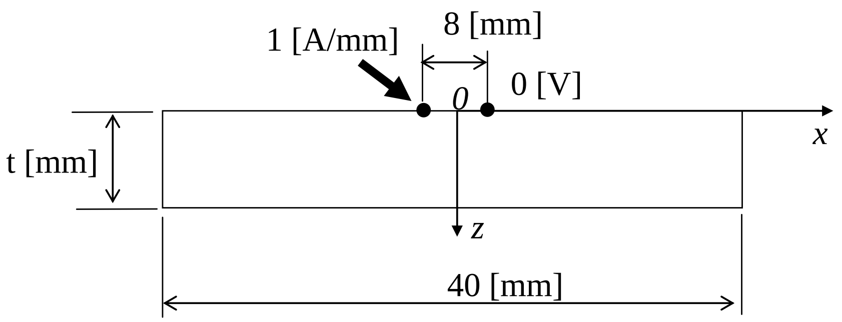
Good results for strongly orthotropic



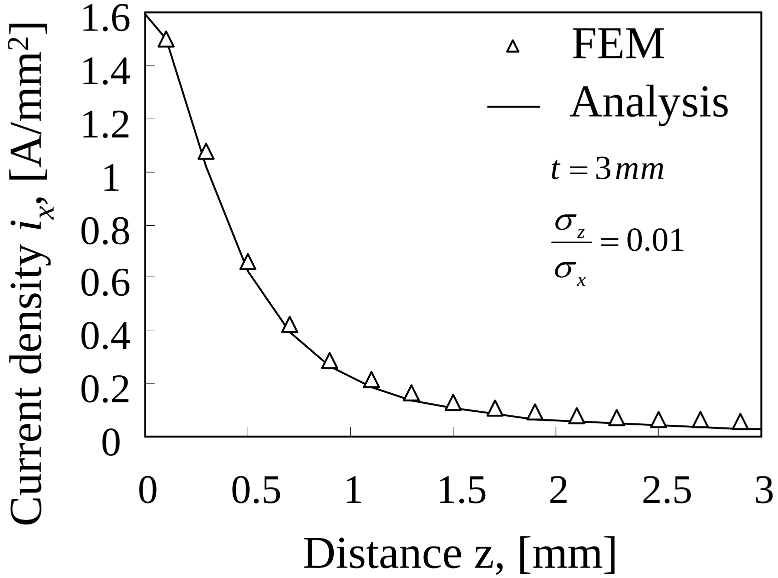


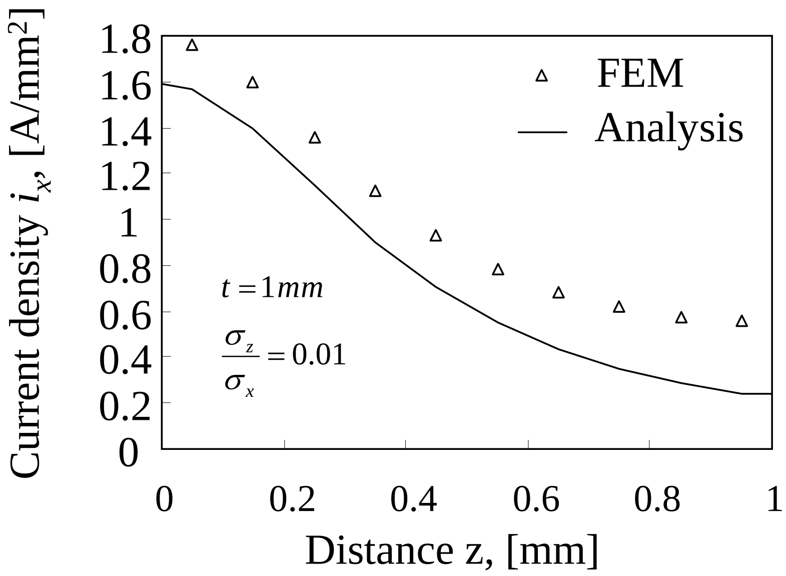


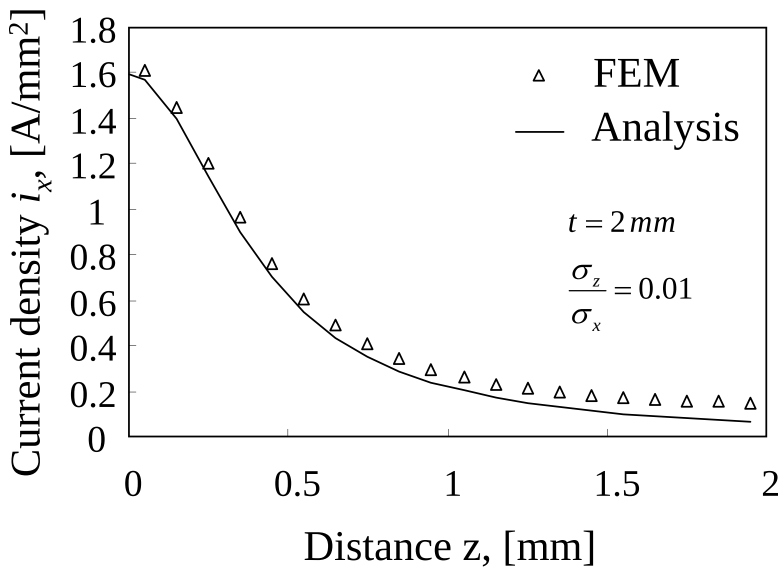
Effect of thickness



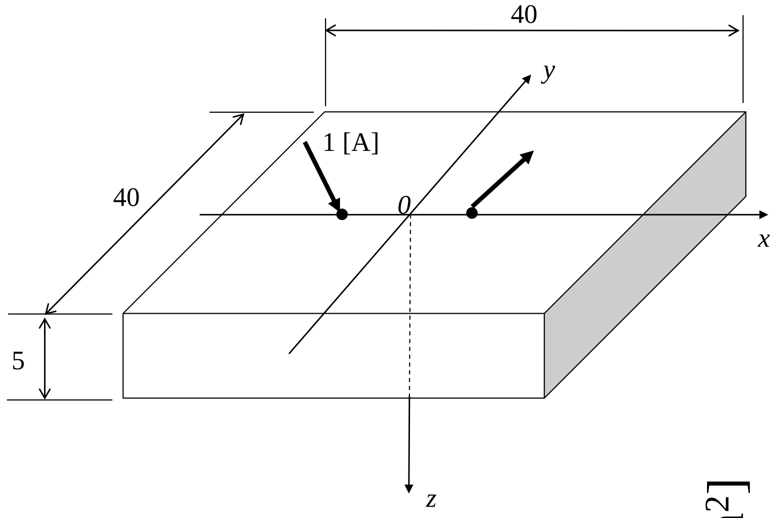
Model B



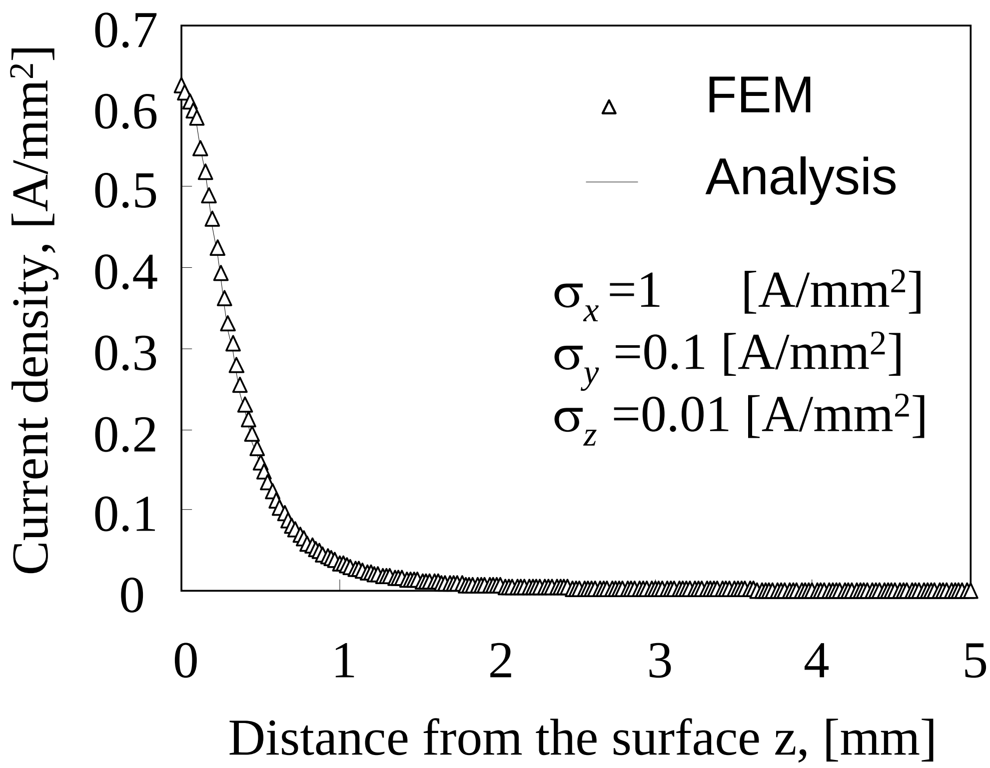




Good results for thick CFRP

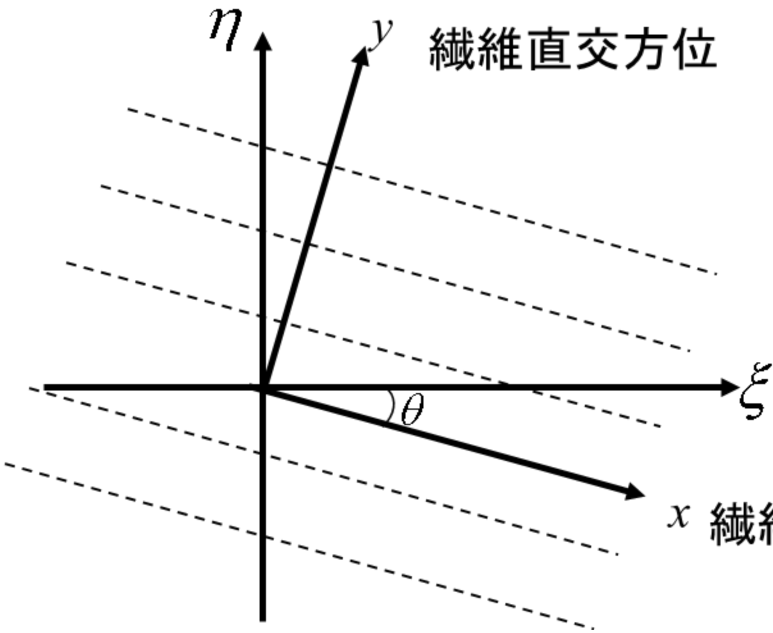

40
y
1 [A]
40
0
x
5
z

Model C


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Current density, [A/mm^2]
FEM
Analysis
σ_x =1 [A/mm^2]
σ_y =0.1 [A/mm^2]
σ_z =0.01 [A/mm^2]
0
1
2
3
4
5
Distance from the surface z, [mm]

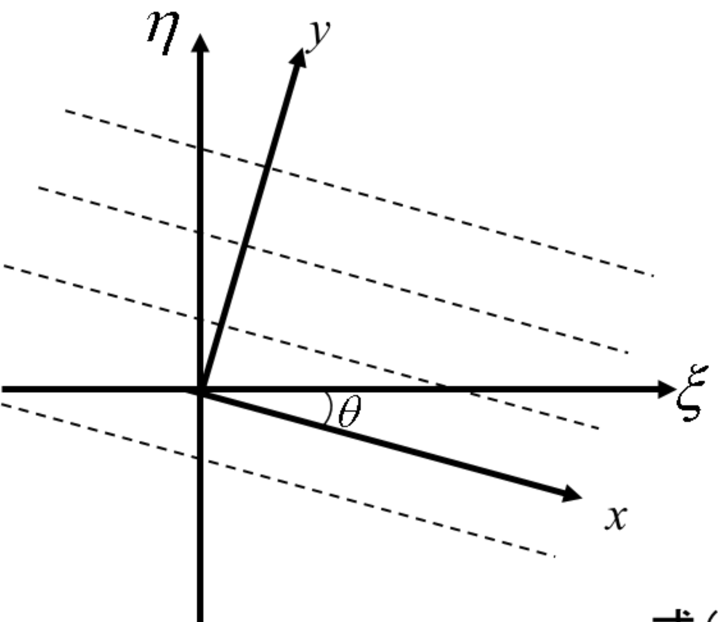
薄板近似（板厚方向に電位差の差異なし）

図のように繊維方位(x-y)座標系から構造座標系(ξーη)座標系への変換を考える



電位差ベクトルの変換は通常のベクトルの回転

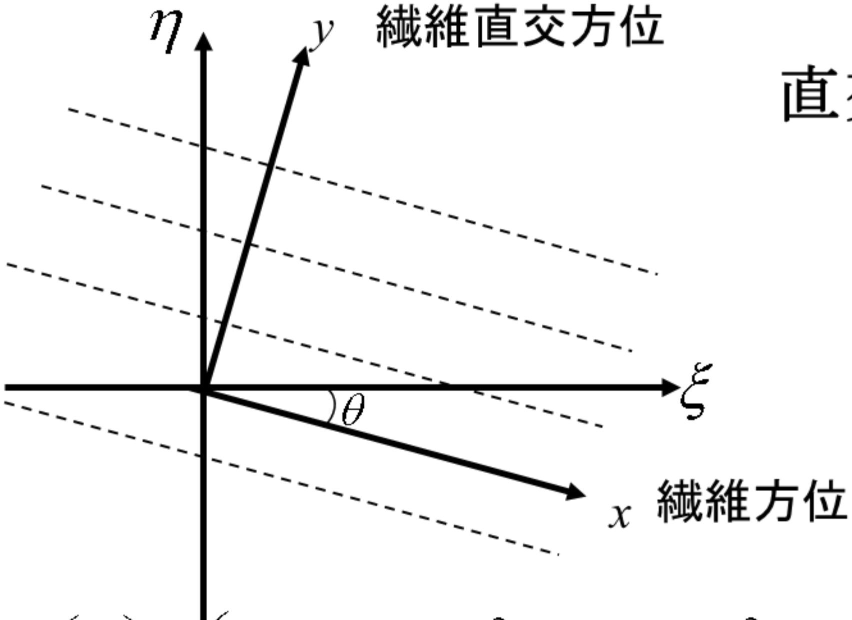
$$\begin{pmatrix} \dfrac{\partial \phi}{\partial \xi} \\ \dfrac{\partial \phi}{\partial \eta} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \dfrac{\partial \phi}{\partial x} \\ \dfrac{\partial \phi}{\partial y} \end{pmatrix}$$



$$\begin{pmatrix} i_\xi \\ i_\eta \end{pmatrix} = \begin{pmatrix} \sigma_x \cos\theta & \sigma_y \sin\theta \\ -\sigma_x \sin\theta & \sigma_y \cos\theta \end{pmatrix} \begin{pmatrix} \dfrac{\partial\phi}{\partial x} \\ \dfrac{\partial\phi}{\partial y} \end{pmatrix} \qquad (3)$$

式(2)から，

$$\begin{pmatrix} \dfrac{\partial\phi}{\partial x} \\ \dfrac{\partial\phi}{\partial y} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{\partial\phi}{\partial\xi} \\ \dfrac{\partial\phi}{\partial\eta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \dfrac{\partial\phi}{\partial\xi} \\ \dfrac{\partial\phi}{\partial\eta} \end{pmatrix} \qquad (4)$$



直交異方性材料の任意方位の導電法則

$$\begin{pmatrix} i_\xi \\ i_\eta \end{pmatrix} = \begin{pmatrix} \sigma_x \cos^2\theta + \sigma_y \sin^2\theta & -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta \\ -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta & \sigma_x \sin^2\theta + \sigma_y \cos^2\theta \end{pmatrix} \begin{pmatrix} \dfrac{\partial\phi}{\partial\xi} \\ \dfrac{\partial\phi}{\partial\eta} \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} \\ \sigma_{\xi\eta} & \sigma_{\eta\eta} \end{pmatrix} \begin{pmatrix} \dfrac{\partial\phi}{\partial\xi} \\ \dfrac{\partial\phi}{\partial\eta} \end{pmatrix}$$

$$\begin{cases} \sigma_{\xi\xi} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta \\ \sigma_{\xi\eta} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta \\ \sigma_{\eta\eta} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta \end{cases}$$

積層理論への拡張

非常に薄い板で板厚方向に板面内方向の電位差分布が存在しない場合
(この仮定は現実には成立しない)

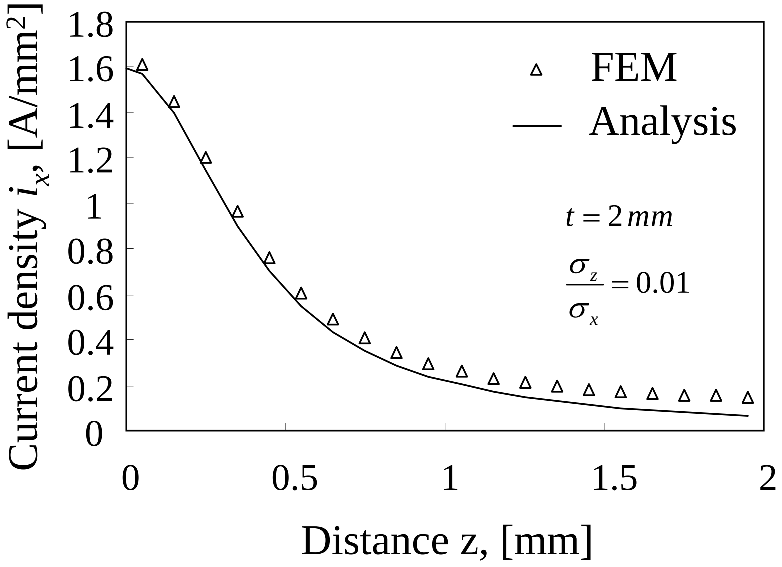
通常の負荷と変形に対する積層理論と同じ導電性に関する積層理論が構築可能である.
面内で電位差が一定であるので，通常の積層理論の面内剛性の場合と同じになり，
積層板の導電性は各層の導電率に各層の体積割合をかけた和となる.

$$\begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} \sum\left(V_0\sigma_0 + V_{90}\sigma_{90} + V_{45}\sigma_{45} + V_{-45}\sigma_{-45}\right) & \sum\left(V_{45}\sigma_{45} + V_{-45}\sigma_{-45}\right) \\ \sum\left(V_{45}\sigma_{45} + V_{-45}\sigma_{-45}\right) & \sum\left(V_0\sigma_0 + V_{90}\sigma_{90} + V_{45}\sigma_{45} + V_{-45}\sigma_{-45}\right) \end{pmatrix} \begin{pmatrix} \dfrac{\partial\phi}{\partial\xi} \\ \dfrac{\partial\phi}{\partial\eta} \end{pmatrix}$$

通常は45°層と-45°層の数が同じ(バランスしている)なので

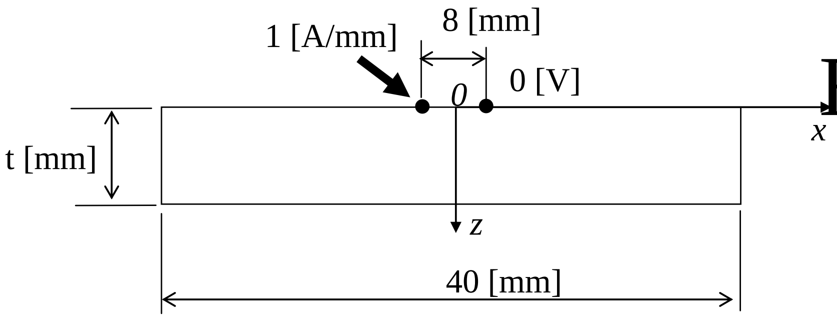
$$\begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} \sum\left(V_0\sigma_0 + V_{90}\sigma_{90} + V_{45}\sigma_{45} + V_{-45}\sigma_{-45}\right) & 0 \\ 0 & \sum\left(V_0\sigma_0 + V_{90}\sigma_{90} + V_{45}\sigma_{45} + V_{-45}\sigma_{-45}\right) \end{pmatrix} \begin{pmatrix} \dfrac{\partial\phi}{\partial\xi} \\ \dfrac{\partial\phi}{\partial\eta} \end{pmatrix}$$

実際にはx方向ポテンシャルの微分はz軸に対して分布している



$$i_x = -\sigma_x \frac{\partial \phi}{\partial x}$$

厚いCFRPでは薄板近似の導電率積層理論は使用できない



Electric current density at x=0

$$i_x = \frac{2Ia\lambda}{\pi\left(z^2 + (a\lambda)^2\right)}$$

$$\lambda = \sqrt{\sigma_z / \sigma_x}$$

Integral of i_x

$$\delta = \int i_x dz = \frac{2}{\pi}\tan^{-1}\left(\frac{z}{a\lambda}\right)$$

Integral value from 0 to t (δ_t)

$$\delta_t = \frac{2}{\pi}\tan^{-1}\left(\frac{t}{a\lambda}\right)$$

$$\frac{t}{a\lambda} \geq 6.3$$

t=2mm δ=0.87,
t=3mm δ=0.91
When δ exceed 0.9,
approximation of infinite plate is applicable

Application to Laminated CFRP

Difficult point

（1） Conductance varies by z

$$\sigma_x \frac{\partial^2 \phi}{\partial x^2} + \sigma_z \frac{\partial^2 \phi}{\partial z^2} = 0 \Rightarrow \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0$$

(ξ is a function of z)

$$I = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} i_r r \cos\theta d\theta r d\varphi$$

Difficult point

(2)How to deal with angles plies

If the laminate comprise of a single angle such as 45º-ply, the laminates can be calculated by rotation of coordinates.

$$i_x = -\sigma_x \frac{\partial \phi}{\partial x} - \sigma_{xy} \frac{\partial \phi}{\partial y}, \ \ i_y = -\sigma_{yx} \frac{\partial \phi}{\partial x} - \sigma_y \frac{\partial \phi}{\partial y}$$

$$\sigma_x \frac{\partial^2 \phi}{\partial x^2} + 2\sigma_{xy} \frac{\partial^2 \phi}{\partial x \partial y} + \sigma_y \frac{\partial^2 \phi}{\partial y^2} = 0$$

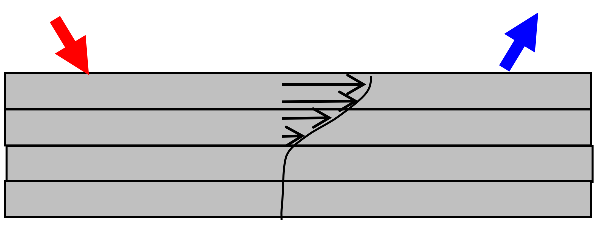
Simple model for laminated CFRP

(1) <u>Cross-ply laminates</u>

(Angle ply is difficult: not orthotropic)

(2) <u>Partial differentiate of potential is the same as that of single ply</u>

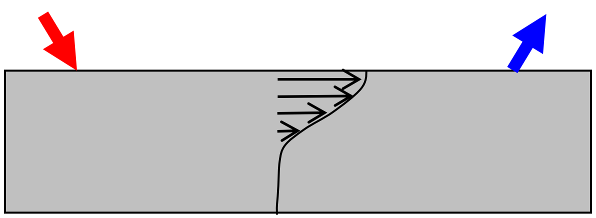
(Obtained from the result of this analysis)



Cross-ply laminated CFRP

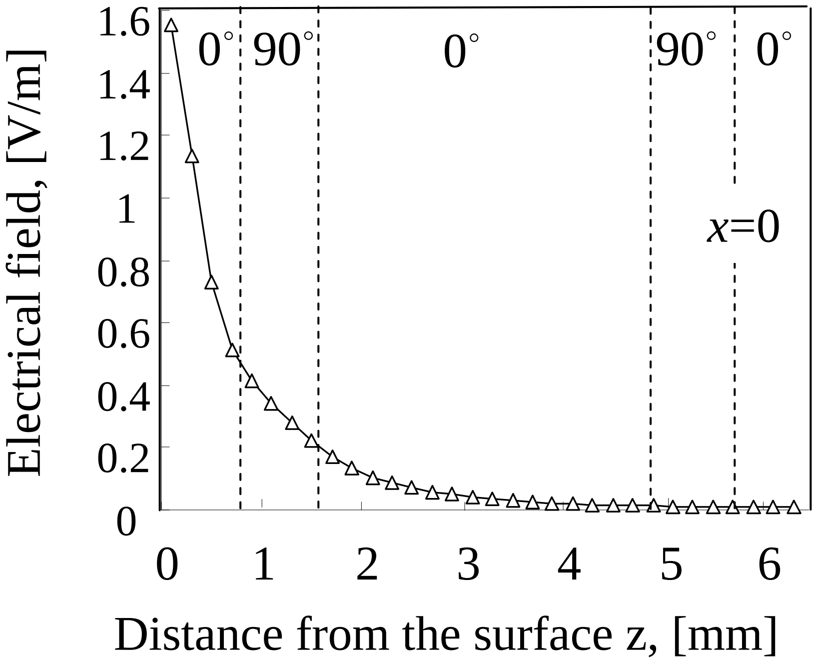
$$\frac{\partial \phi}{\partial x}$$

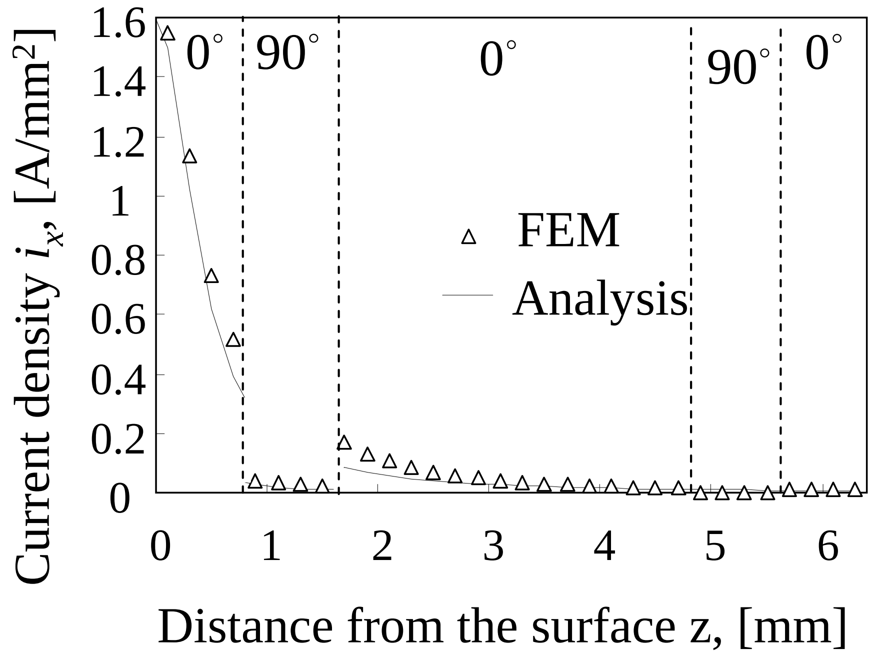
Same



Unidirectional thick CFRP

Cross-ply laminate



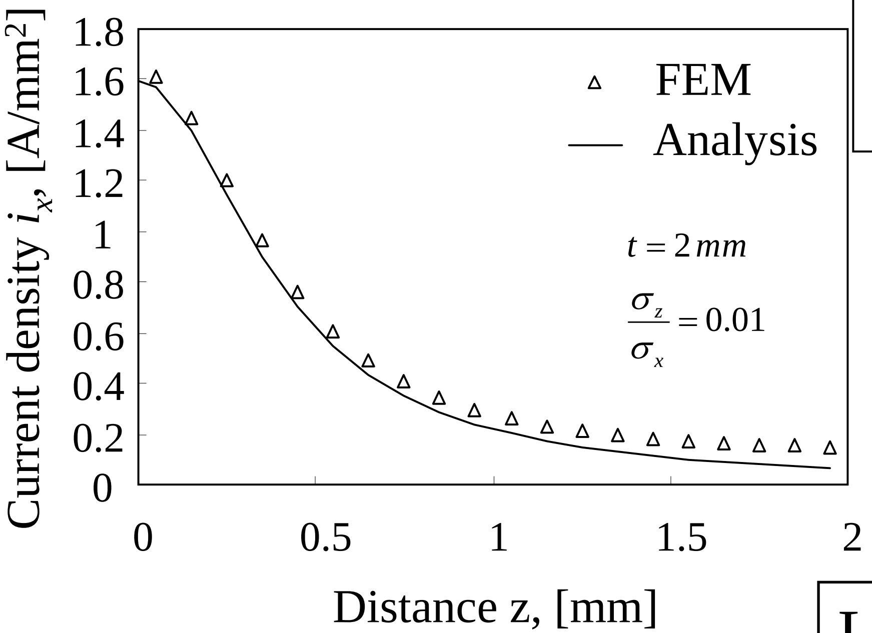


$\dfrac{\partial \phi}{\partial x}$ —— Calculated

△ FEM

Good estimations

New lamination theory for thick CFRP



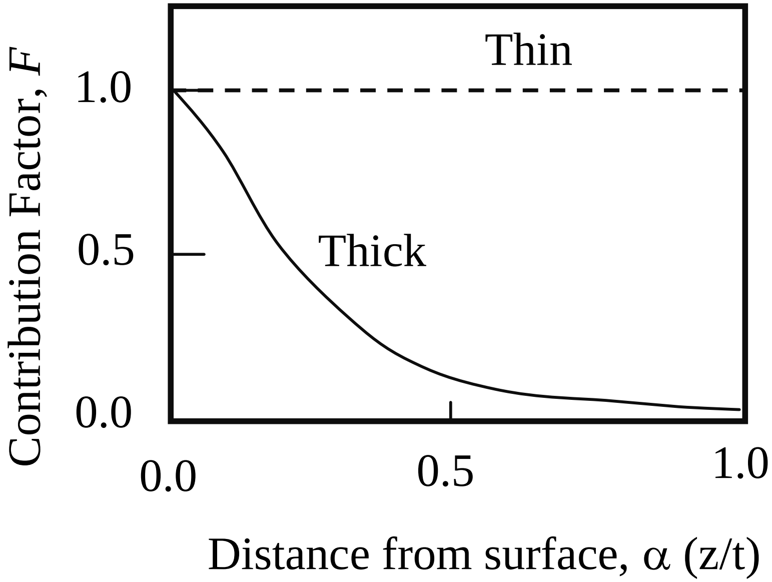
$$i_x = -\sigma_x \frac{\partial \phi}{\partial x}$$

Difference of the potential in the *x*- direction has distribution along the z-axis

Thin plate theory is not applicable (Most of electric current flows near the surface)

Large errors are expected when an entire aircraft is calculated using shell type elements

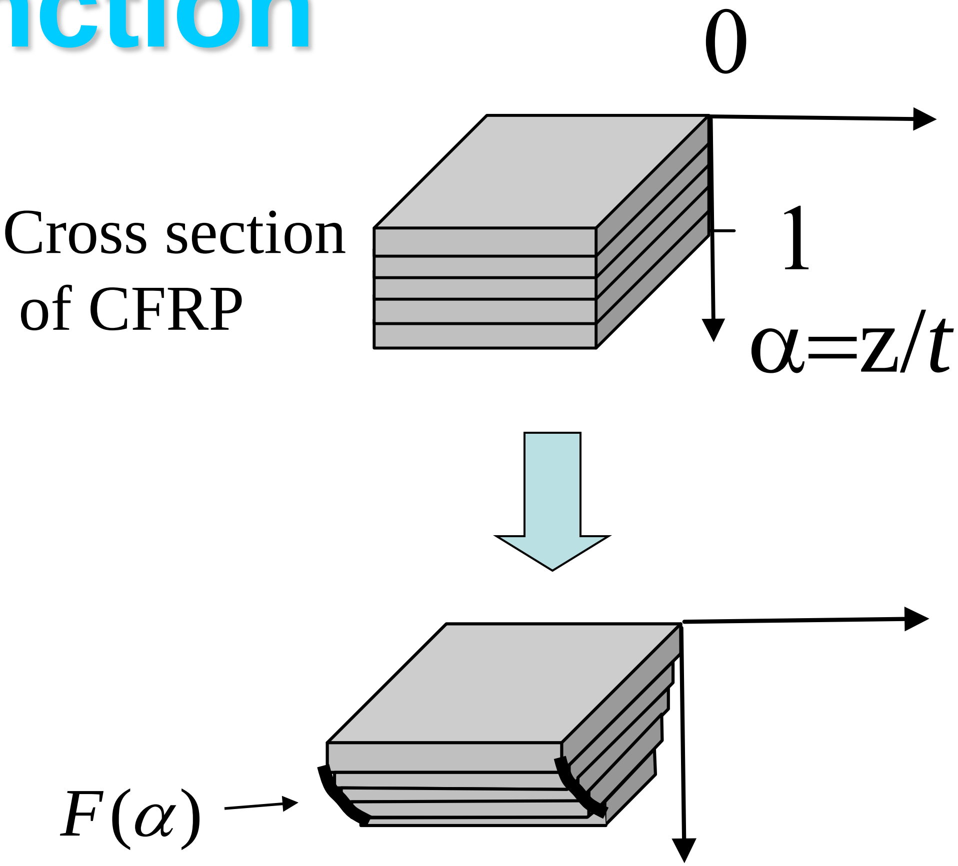
Contribution Function



$$C_{\xi\xi} = \int_0^1 F(\alpha)\sigma_{\xi\xi}\,d\alpha = \sum_{k=1}^{N} \frac{t_p}{t} F_k \sigma_{\xi\xi}^k$$

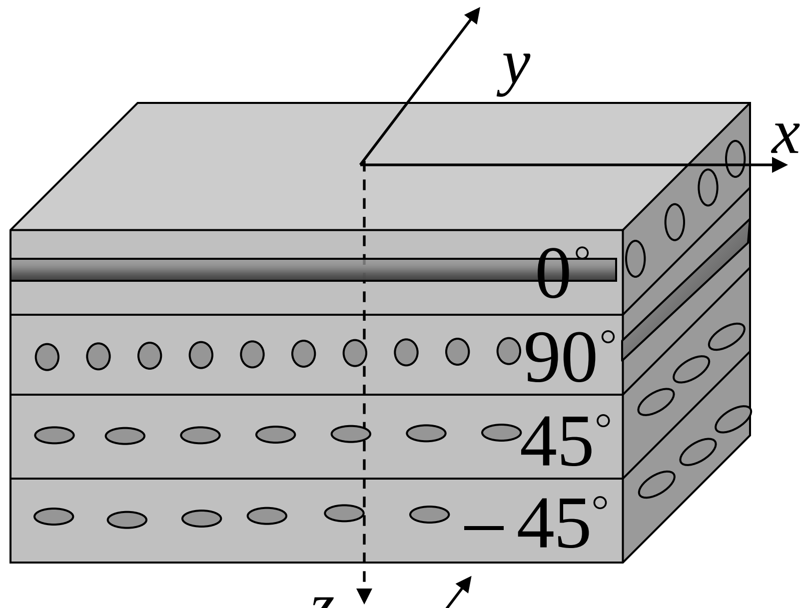
$$C_{\xi\eta} = \int_0^1 F(\alpha)\sigma_{\xi\eta}\,d\alpha = \sum_{k=1}^{N} \frac{t_p}{t} F_k \sigma_{\xi\eta}^k$$

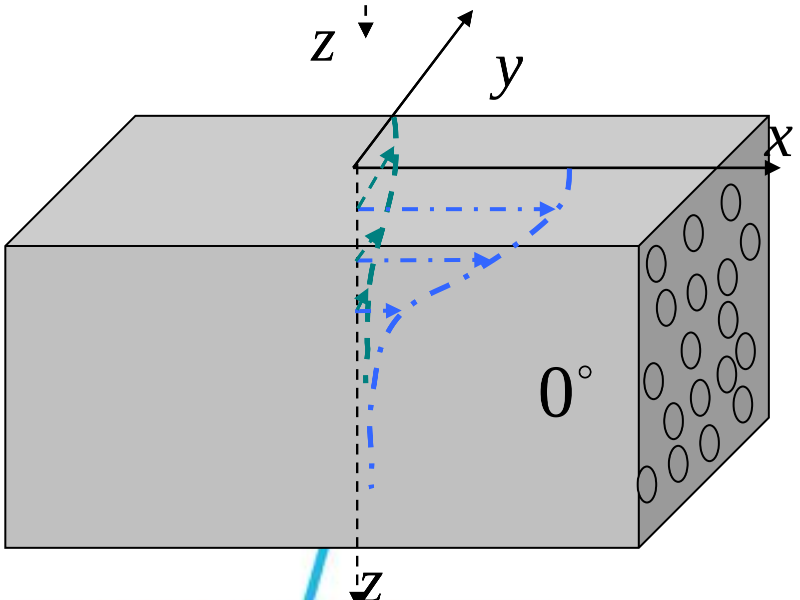
$$C_{\eta\eta} = \int_0^1 F(\alpha)\sigma_{\eta\eta}\,d\alpha = \sum_{k=1}^{N} \frac{t_p}{t} F_k \sigma_{\eta\eta}^k$$



$F(\alpha)$ gives the decrease of cross sectional area for electric current

How to obtain the Contribution Function $F(\alpha)$





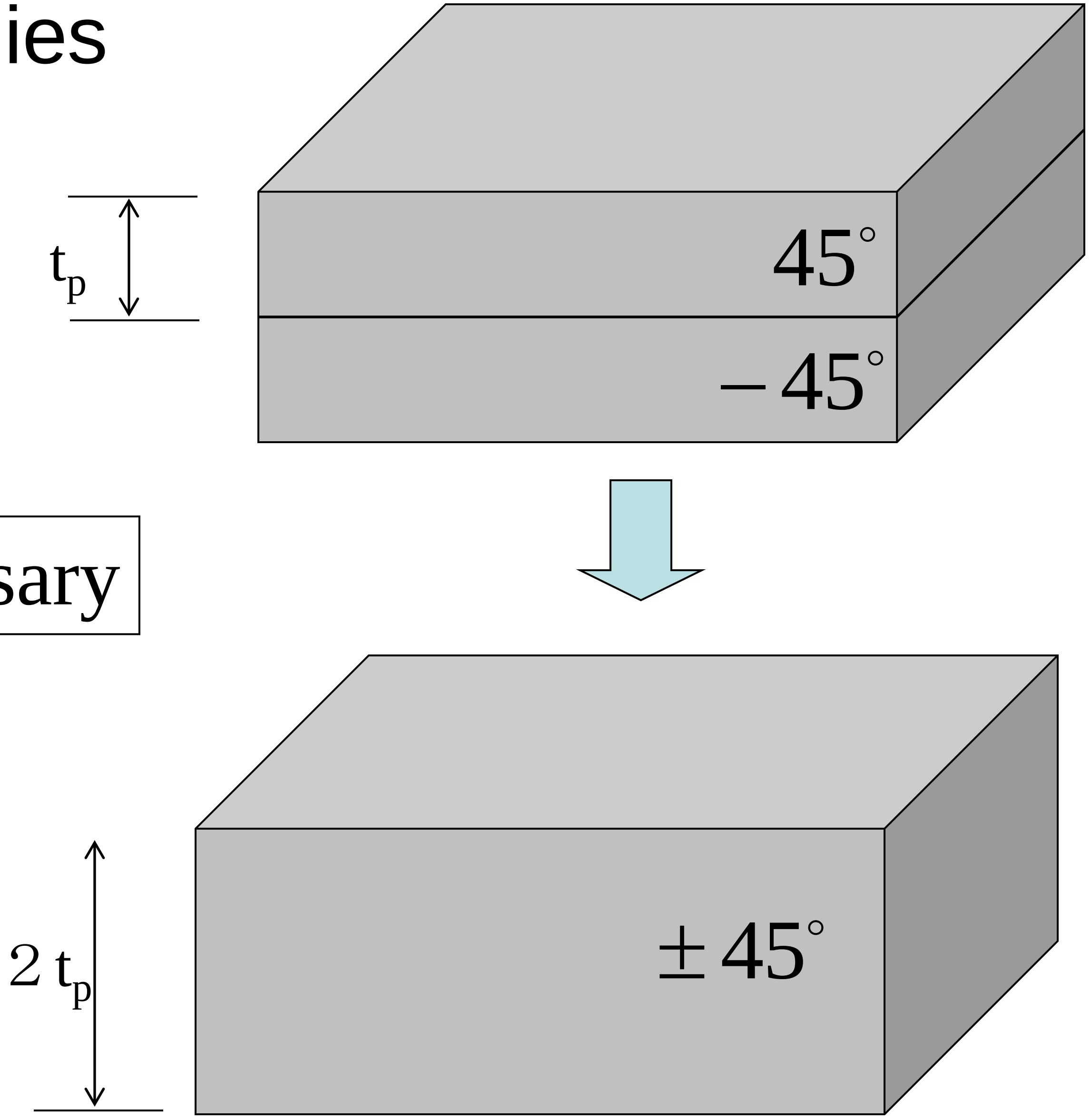
(1)Analysis of thick unidirectional CFRP gives $\partial\phi/\partial x$. From this result, we can obtain $F_x(\alpha)$

(2)Analysis of thick unidirectional CFRP gives $\partial\phi/\partial y$. From this result, we can obtain $F_y(\alpha)$

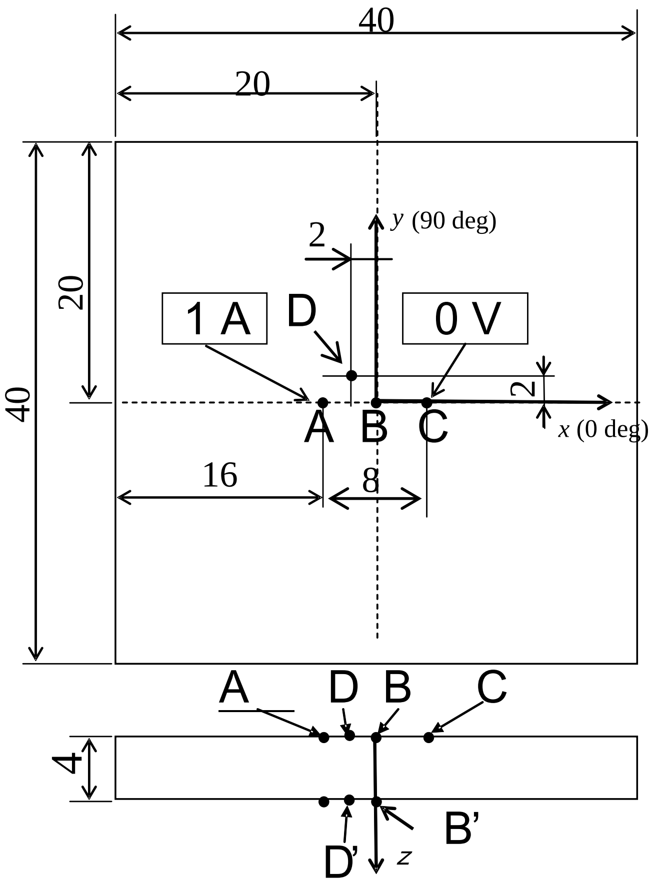
(3)We cannot obtain $F_{xy}(\alpha)$ because the results of the thick unidirectional CFRP has no electric current due to the interaction term.

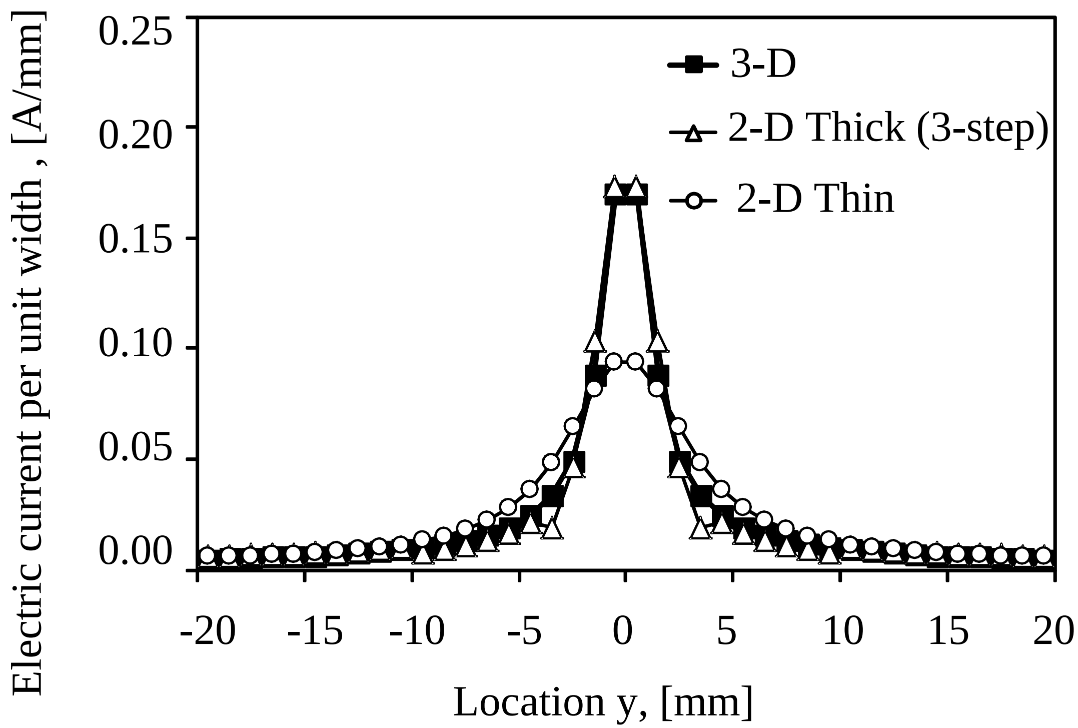
Coupling of $\pm 45^\circ$ plies

F_{xy}(a) is not necessary



Comparison with 3D FEM analysis





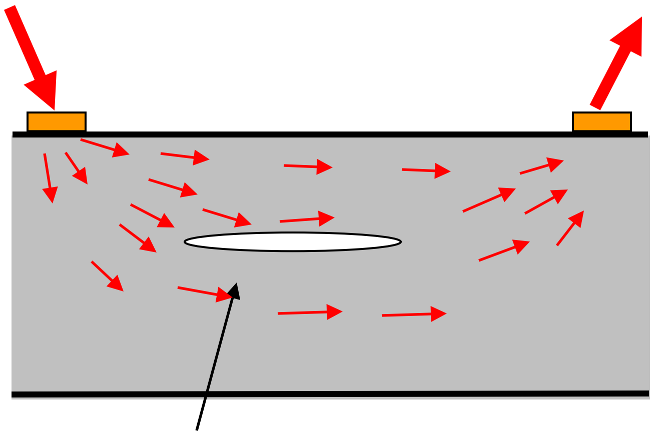
Comparison of the number of elements

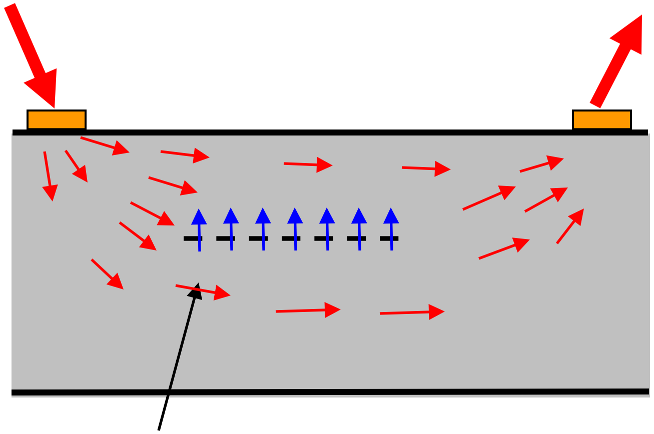
1,600/286,720＝0.56%

Error 4%

ポテンシャル解析による今後

- はく離の影響を評価可能





２重湧き出しを設置することではく離面に垂直な電流をキャンセルして解析可能

たくさんの実験やFEM解析なしではく離位置と大きさの同定が可能となる可能性がある

$D_x = \varepsilon_x E_x,\ D_y = \varepsilon_y E_y,\ D_z = \varepsilon_z E_z$

表皮効果

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_z}{\partial t} = 0$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \frac{\partial D_x}{\partial t} = i_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \frac{\partial D_y}{\partial t} = i_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \frac{\partial D_z}{\partial t} = i_z$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

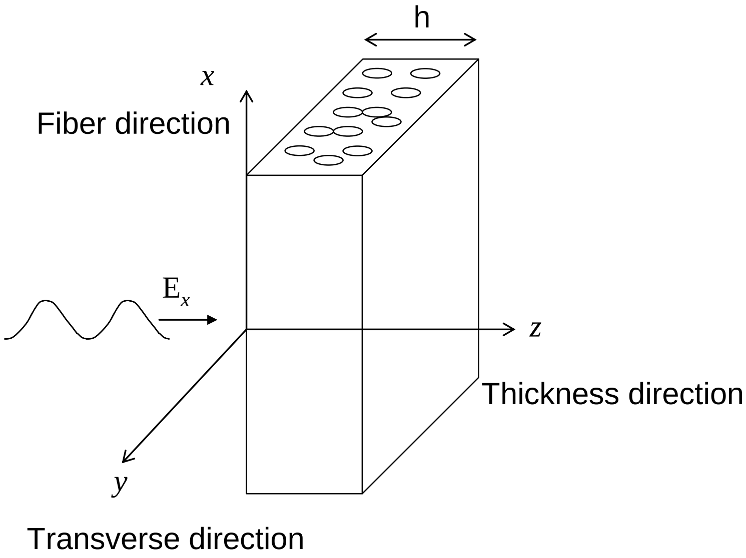
$$B_x = \mu_x H_x,\ B_y = \mu_y H_y,\ B_z = \mu_z H_z$$

$$D_x = \varepsilon_x E_x,\ D_y = \varepsilon_y E_y,\ D_z = \varepsilon_z E_z$$

$$\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}=\mu_0\sigma_x\frac{\partial E_x}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial z^2}=\mu_0\sigma_y\frac{\partial E_y}{\partial t}$$

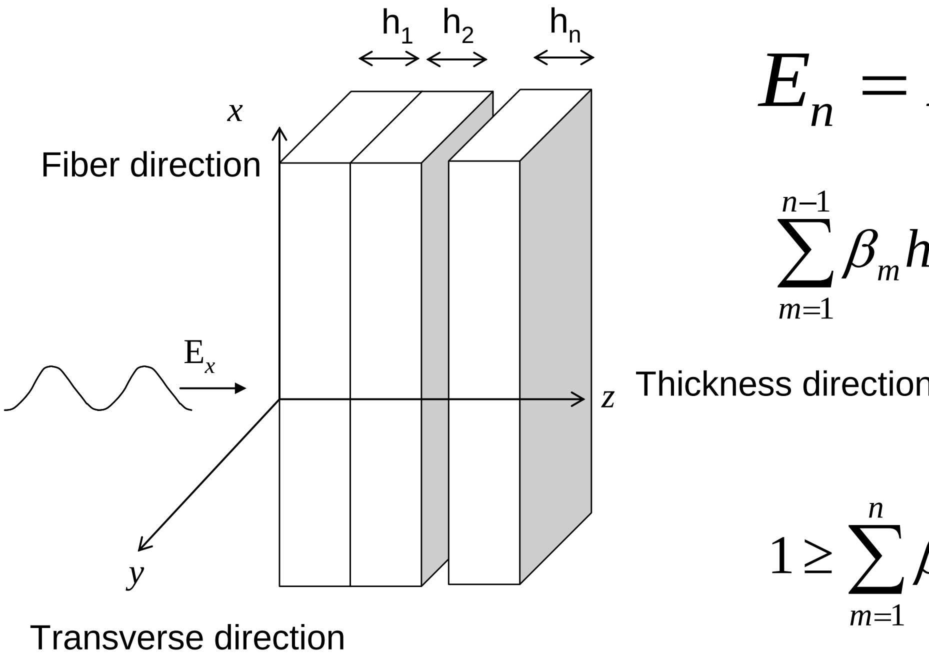
$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\frac{\partial^2 E_z}{\partial z^2}=\mu_0\sigma_z\frac{\partial E_z}{\partial t}$$



$$\frac{\partial^2 E_x}{\partial z^2}=\mu_0\sigma_x\frac{\partial E_x}{\partial t}$$

$$E_x=E_0 e^{-jwt}e^{-kz}$$

$$\delta=\sqrt{\frac{2}{\mu_0\sigma_x\varpi}}$$



$$E_n = E_0 e^{-\beta_1 h_1} e^{-\beta_2 h_2} \cdots e^{-\beta_{n-1} h_{n-1}}$$

$$\sum_{m=1}^{n-1} \beta_m h_m + \beta_n \left(\delta_n - \sum_{m=1}^{n-1} h_m \right) = 1$$

$$1 \geq \sum_{m=1}^{n} \beta_m h_m = \sqrt{\frac{\mu_0 \omega}{2}} \sum_{m=1}^{n} \sqrt{\sigma_{xm}} h_m$$

$$\delta_n \approx \sum_{m=1}^{n} h_m$$

異方性による直流電流の表皮効果

$$i_x = \frac{I}{\pi\sqrt{\sigma_x\sigma_z}}\left\{\frac{x+a}{\frac{(x+a)^2}{\sigma_x}+\frac{z^2}{\sigma_z}} - \frac{x-a}{\frac{(x-a)^2}{\sigma_x}+\frac{z^2}{\sigma_z}}\right\}$$

$$i_x = \frac{2aI\sqrt{\sigma_x\sigma_z}}{\pi\left(\sigma_z a^2 + \sigma_x z^2\right)}$$

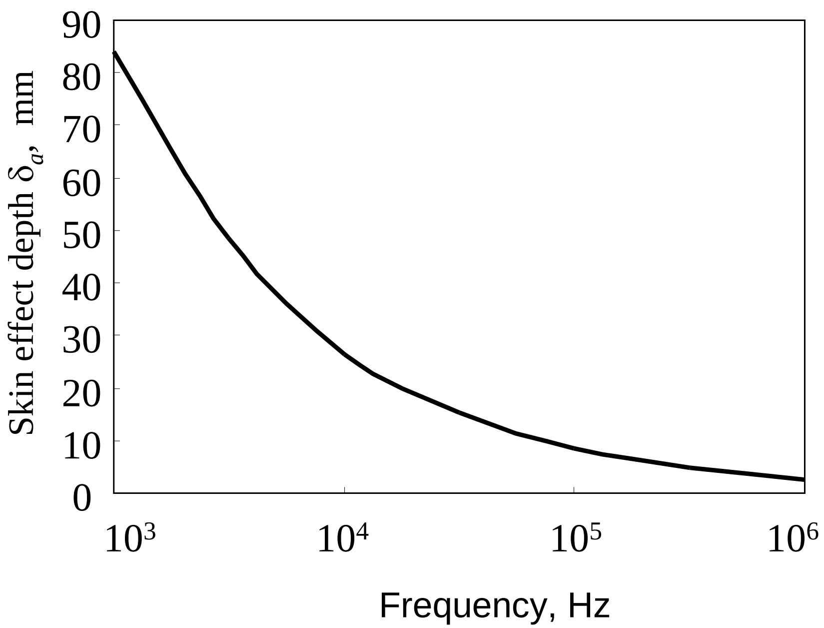
$$\delta_d = \sqrt{e-1}\sqrt{\frac{\sigma_z}{\sigma_x}}a \approx 1.311\sqrt{\frac{\sigma_z}{\sigma_x}}a$$

IM600/133 $\delta_d = 0.3a\times 10^{-3}\ [m]$

$2a$=0.5m, 1m, 2m

δd＝0.075mm, 0.15mm, 0.3mm

交流電流の表皮効果厚さ



雷撃損傷などは直流電流解析で近似可能

6. 結　論

(1) 直交異方性の導電率を示し，座標変換により
完全流体のポテンシャル流で解析可能であること
を示した.

(2)直交積層においては，ポテンシャル流で電流密度
が解析可能である

(3)寄与度関数を用いた解析により．厚板CFRPを2
次元解析する導電率の積層理論を提案し，その有
効性を示した.

（4）交流表皮効果を解析し，CFRPでは異方性の効果
の方が大きいことを示した.