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SELF-ORGANIZING MAP OF PARETO SOLUTIONS OBTAINED FROM MULTIOBJECTIVE SUPERSONIC WING DESIGN

Shigeru Obayashi* and Daisuke Sasaki**
Tohoku University
Katahira 2-1-1-, Sendai, Japan 980-8577

Abstract

Self-Organizing Map (SOM) has been applied to analyze 766 Pareto solutions obtained from the four-objective aerodynamic optimization of supersonic wings using Evolutionary Algorithms. Three-dimensional Pareto front (tradeoff surface) is mapped onto the two-dimensional SOM where global tradeoffs are successfully visualized. Furthermore, from the clusters obtained in the SOM, the design variables are mapped onto another SOM. This leads to clusters of design variables which indicate the relative importance of design variables and their interactions. SOM is confirmed to be a versatile datamining tool for aeronautical engineering.

1. Introduction*

Multiobjective Evolutionary Algorithms (MOEAs) are getting popular in many fields because they will provide a unique opportunity to address global tradeoffs between multiple objectives by sampling a number of Pareto solutions. Especially in the field of aeronautical engineering, a series of study for aerodynamic design of supersonic wings has been performed by the present authors[1-3]. In the latest report[3], four design objectives were used and the resulting Pareto front was obtained as a three-dimensional surface in the four-dimensional objective function space. Although 766 Pareto solutions were obtained in total, only a few solutions were examined in detail. This is a typical case that computer produces/accumulates too much data. To make a good use of the large data, datamining techniques are needed.

One of the popular datamining techniques is the Self-Organizing Map (SOM) by Kohonen[4,5]. The SOM is one of neural network models. The SOM algorithm is based on unsupervised, competitive learning. It provides a topology preserving mapping from the high dimensional space to map units. Map units, or neurons, usually form a two-dimensional lattice and thus the mapping is a mapping from high dimensional space onto a plane. The property of topology preserving means that the mapping preserves the relative distance between the points. Points that are near each other in the input space are mapped to nearby map units in the SOM. The SOM can thus serve as a cluster analyzing tool of high-dimensional data.

In this paper, the SOM is applied to map Pareto solutions obtained in [3]. This will reveal the global tradeoffs between four design objectives. Furthermore, from the clusters obtained in the SOM, the relations between design variables are mapped onto another SOM. This will indicate the relative importance of design variables and their interactions.

2. Multiobjective Aerodynamic Optimization

2.1 Formulation of Four-Objective Optimization

The objective functions used in [3] can be stated as follows:

1. Drag coefficient at transonic cruise, $C_{D,t}$
2. Drag coefficient at supersonic cruise, $C_{D,s}$
3. Bending moment at the wing root at supersonic cruise condition, M_B
4. Pitching moment at supersonic cruise condition, M_P

In the present optimization, all four objective functions are to be minimized. The transonic drag minimization corresponds to the cruise over land, the supersonic drag minimization corresponds to the cruise over sea. Lower bending moments allow less structural weight to support the wing. Lower pitching moments mean less trim drag.

The present optimization is performed at two design points for the transonic and supersonic cruises. Corresponding flow conditions and the target lift coefficients are described as

1. Transonic cruising Mach number, $M_{\infty,t} = 0.9$
2. Supersonic cruising Mach number, $M_{\infty,s} = 2.0$
3. Target lift coefficient at transonic cruising condition, $C_{L,t} = 0.15$
4. Target lift coefficient at supersonic cruising condition, $C_{L,s} = 0.10$

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*Associate Professor, Institute of Fluid Science, Associate Fellow
**Graduate Student, Institute of Fluid Science, Student Member

5. Reynolds number based on the root chord length at both conditions, Re = 1.0 x 10^7

Flight altitude is assumed at 10 km for the transonic cruise and at 15 km for the supersonic cruise. To maintain lift constraints, the angle of attack is computed for each configuration by using $C_{L\alpha}$ obtained from the finite difference. Thus, three Navier-Stokes computations per evaluation are required. During the aerodynamic optimization, wing area is frozen at a constant value.

Design variables are categorized to planform, airfoil shapes and the wing twist. The wing planform is determined by six design variables as shown in Fig. 1. A chord length at the wing tip is determined accordingly because of the fixed wing area. Airfoil shapes are composed of its thickness distribution and camber line. The thickness distribution is represented by a Bézier curve defined by nine polygons. The wing thickness is constrained for structural strength. The thickness distributions are defined at the wing root, kink and tip, and then linearly interpolated in the spanwise direction. The total number of polygons is 33 for the entire thickness distribution.

The camber surfaces composed of the airfoil camber lines are defined at the inboard and outboard of the wing separately. Each surface is represented by the Bézier surface defined by four polygons in the chordwise direction and three in the spanwise direction. The number of polygons that defines two camber surfaces is 20 in total. Finally, the wing twist is represented by a B-spline curve with six polygons. As a result, 72 design variables are used to define a whole wing shape. In Fig. 2, a three-dimensional wing with computational structured grid is illustrated. See [3] for details.

2.2 Overview of Pareto Solutions

The evolution was computed for 75 generations. After the computation, all the solutions evolved were sorted again to find the final Pareto solutions. The Pareto solutions were obtained in the four-dimensional objective function space. To understand the distribution of Pareto solutions in a conventional manner, all Pareto solutions are projected into the two-dimensional objective function space between transonic and supersonic drag coefficients as shown in Fig. 3. In Fig. 3, Surface I shows the tradeoff between aerodynamic performances. The wings near Surface I have impractically large aspect ratios. The planform shapes of the extreme Pareto solutions that minimize the respective objective functions appear physically reasonable as shown in Fig. 4. A wing with the minimal transonic cruising drag has a less leading-edge sweep and a large aspect ratio. On the contrary, a wing with the lowest supersonic drag coefficient has a large leading-edge sweep to remain inside the Mach cone. The pitching moment is reduced by lowering the sweep angle and the wing chord length.

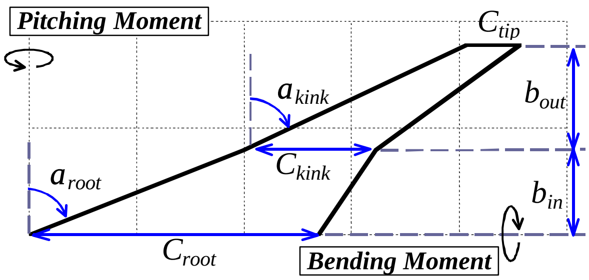


Fig. 1 Wing planform definition

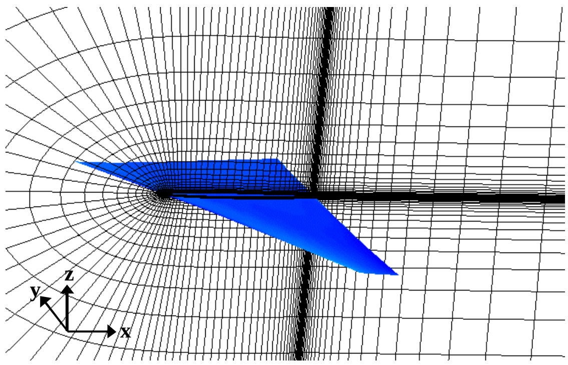


Fig. 2. Wing with structured grid in C-H topology

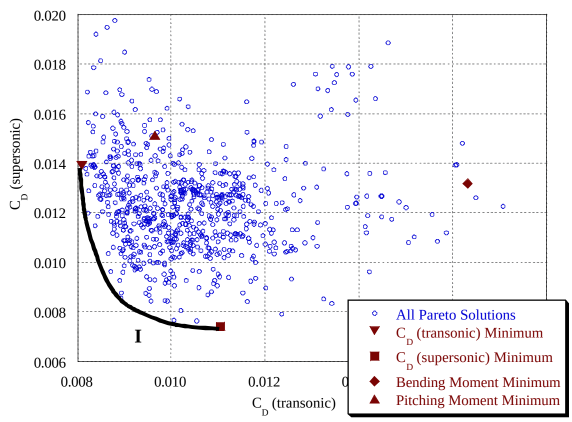


Fig. 3 Projection of Pareto solutions into two-dimensional plane between transonic and supersonic drag coefficients

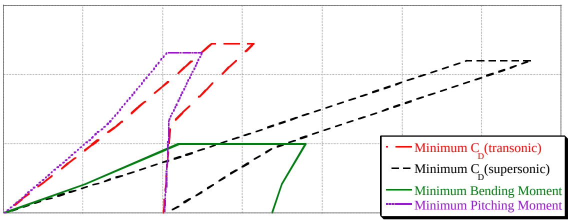
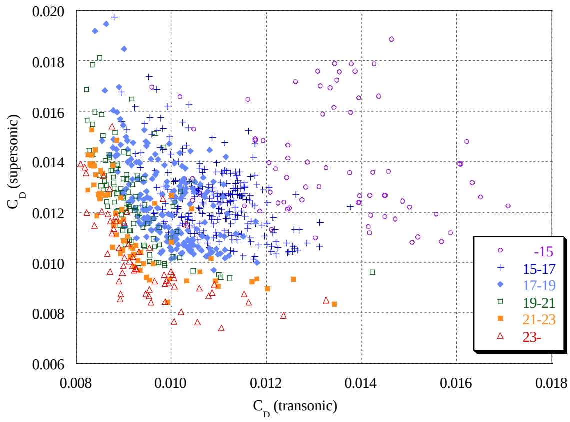
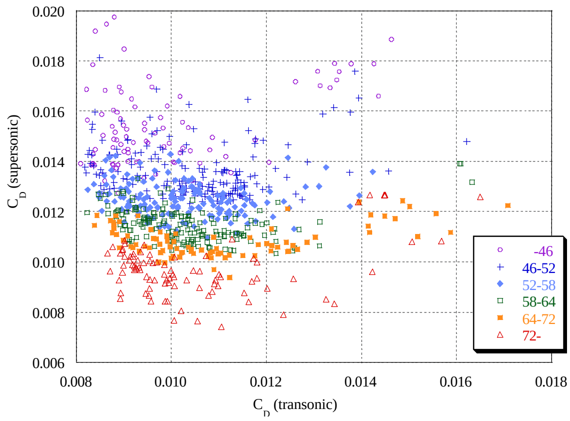


Fig. 4 Planform shapes of the extreme Pareto solutions

Figure 3 contains 766 Pareto solutions. However, it is difficult to pick up particular solutions unless they are extreme solutions as indicated in Fig. 4. To visualize the solutions further, all the Pareto solutions in Fig. 3 are labeled by the bending and pitching moments, respectively, and plotted in Fig. 5. The wings near the tradeoff surface between transonic and supersonic drag coefficients (tradeoff surface I in Fig. 3) have impractically large bending moments as shown in Fig. 5 (a). The bending moment is closely related to both transonic and supersonic drag coefficients. On the other hand, the pitching moment has an influence only on supersonic drag coefficient as seen in Fig. 5 (b). However, these figures only demonstrate a facet of the Pareto front. Therefore, the SOM will be introduced in the next section.



(a) Labeled according to bending moment



(b) Labeled according to pitching moment

Fig. 5 Projection of Pareto front to supersonic and transonic drag tradeoffs.

3. SOM

3.1 Neural Network and SOM

The SOM [4,5] is a two-dimensional array of neurons:

$$\mathbf{M} = \{\mathbf{m}_1 \quad \cdots \quad \mathbf{m}_{p\times q}\}$$

This is illustrated in Fig. 6. One neuron is a vector called the codebook vector

$$\mathbf{m}_i = [m_{i_1} \quad \cdots \quad m_{i_n}]$$

This has the same dimension as the input vectors (n-dimensional). The neurons are connected to adjacent neurons by a neighborhood relation. This dictates the topology, or the structure, of the map. Usually, the neurons are connected to each other via rectangular or hexagonal topology. In Fig. 6 the topological relations are shown by lines between the neurons.

One can also define a distance between the map units according to their topology relations. Immediate neighbors (the neurons that are adjacent) belong to the neighborhood N_c of the neuron $\mathbf{m}_c$. The neighborhood function should be a decreasing function of time: $N_c = N_c(t)$.

The training consists of drawing sample vectors from the input data set and "teaching" them to the SOM. The teaching consists of choosing a winner unit by means of a similarity measure and updating the values of codebook vectors in the neighborhood of the winner unit. This process is repeated a number of times.

In one training step, one sample vector is drawn randomly from the input data set. This vector is fed to all units in the network and a similarity measure is calculated between the input data sample and all the codebook vectors. The best-matching unit is chosen to be the codebook vector with greatest similarity with the input sample. The similarity is usually defined by means of a distance measure. For example in the case of Euclidean distance the best-matching unit is the closest neuron to the sample in the input space.

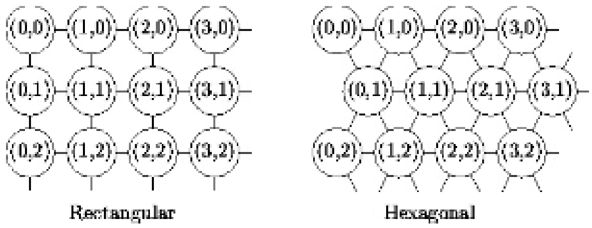


Fig. 6 Different topologies used in the SOM

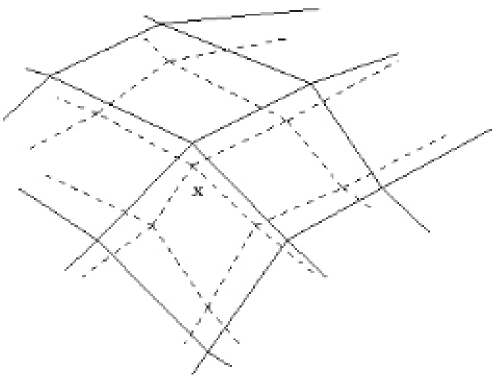


Fig. 7 Updating the best matching unit and its neighbors

The best-matching unit, usually noted as $\mathbf{m}_c$, is the codebook vector that matches a given input vector $\mathbf{x}$ best. It is defined formally as the neuron for which

$$\|\mathbf{x}-\mathbf{m}_c\| = \min_i [\|\mathbf{x}-\mathbf{m}_i\|]$$

After finding the best-matching unit, units in the SOM are updated. During the update procedure, the best-matching unit is updated to be a little closer to the sample vector in the input space. The topological neighbors of the best-matching unit are also similarly updated. This update procedure stretches the best-matching unit and its topological neighbors towards the sample vector.

In Fig.7, the update procedure is illustrated. The codebook vectors are situated in the crossings of the solid lines. The topological relationships of the SOM are drawn with lines. The input fed to the network is marked by $\mathbf{x}$ in the input space. The best-matching unit, or the winner neuron is the codebook vector closest to the sample, in this example the codebook vector in the middle above $\mathbf{x}$. The winner neuron and its topological neighbors are updated by moving them a little towards the input sample. The neighborhood in this case consists of the eight neighboring units in the figure. The updated network is shown in the same figure with dashed lines. In the following, SOMs were generated in the hexagonal topology by using Viscovery® SOMine 3.0J [6]

3.2 SOM of Objective Function Space

The Pareto solutions for supersonic wing designs obtained in [3] have four design objectives. First, let's project this four-dimensional objective function space onto the two-dimensional SOM. Figure 8 shows the resulting SOM with seven clusters. For better understanding, the typical planform shapes are plotted in the figure. Lower right corner of the map corresponds to highly swept, high aspect ratio wings good for supersonic aerodynamics. Lower left corner corresponds to moderate sweep angles good for reducing the pitching moment. Upper right corner corresponds to small aspect ratios good for reducing the bending moment. Upper left corner thus reduces both pitching and bending moments.

Figure 9 shows the same SOM colored by four design objective values, aspect ratios and taper ratios. Low transonic drag region corresponds to high aspect ratio region, which is reasonable for subsonic wings in general. Low transonic drag region also corresponds to low taper ratio, although the relation appears slightly fuzzy. This is also reasonable because a tapered wing will likely give a proper elliptic spanwise loading. Low supersonic drag region corresponds to high pitching moment region. This is primarily because of high sweep angles. Low supersonic drag region also corresponds to high bending moment region. Combination of high sweep angle and high aspect ratio results in very large bending moment. These confirm that supersonic wing design is highly constrained.

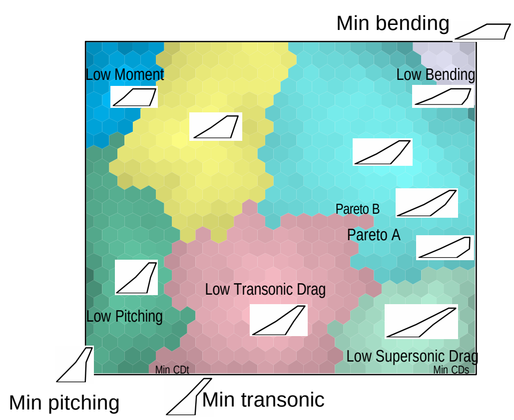
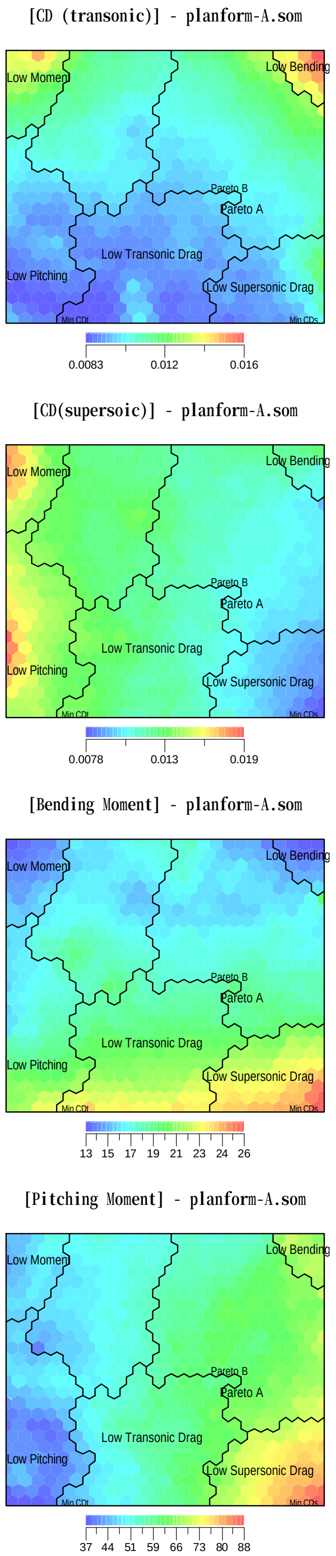


Fig. 8 SOM of Pareto solutions in the objective function space



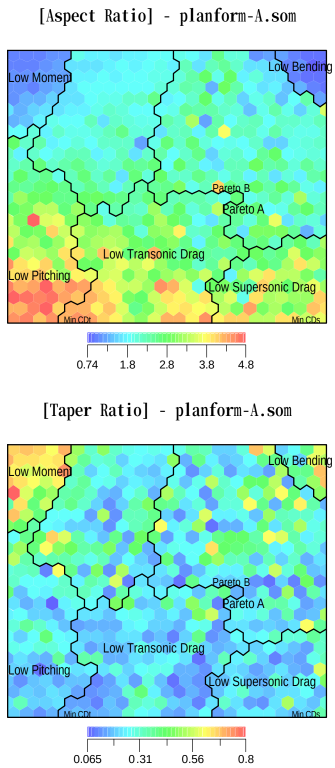


Fig. 9 SOM colored by each design objectives, aspect ratios and taper ratios

3.3 SOM of Design Variable Space

Design is a process to find a point in the design variable space that matches with the given point in the objective function space. This is, however, very difficult. For example, the present design variable space has 72 dimensions. One way of overcoming high dimensionality is to group some of design variables together. To do so, the cluster analysis based on SOM can be applied.

First, the previous SOM is revised by using small clusters in total of 48 as shown in Fig. 10. Then, all the design variables are averaged in every cluster, respectively. Now each design variable has a codebook vector of cluster-averaged values. Finally, a new SOM is generated from the cluster-averaged data as shown in Fig. 11.

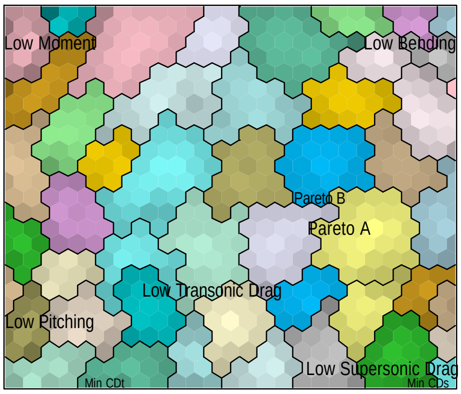


Fig. 10 SOM of objective function space with 48 clusters

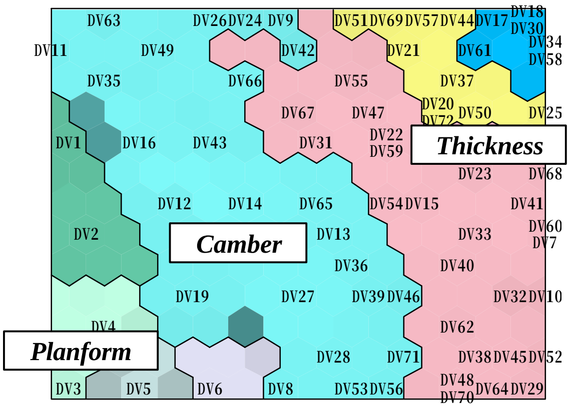


Fig. 11 SOM of design variable space

In Fig. 11, the labels indicate 72 design variables. DV1 to 6 corresponds to the planform design variables. These variables have dominant influence on the wing performance. DV1 and 2 determine the span lengths of the two wing panels and make a cluster together. DV3 and 4 correspond to leading-edge sweep angles and make another cluster. DV5 and 6 are root-side chord lengths.

DV7 to 26 defines wing camber. Some of these variables appear next to the planform variables and this is consistent with aerodynamic knowledge that wing camber is essential for aerodynamic performance. DV27 to 33 determine wing twist.

DV34 to 72 are design variable for wing thickness. These design variables only appear in the map from the middle region to right. This corresponds to the wing theory where the wing thickness is often ignored. Aerodynamic performance of a wing is primarily determined by its planform shape, wing camber and twist.

The design variable space with 72 dimensions is mapped onto two-dimensional SOM, where aerodynamic knowledge can be applied to understand the characteristics of the wing design. Although the present neural network does not know aerodynamics, the resulting SOM is confirmed to perform aerodynamic datamining properly.

4. Concluding Remarks

Self-Organizing Map has been applied to analyze 766 Pareto solutions obtained from the previous four-objective aerodynamic optimization of supersonic wings using Evolutionary Algorithms. Three-dimensional Pareto front is mapped onto the two-dimensional SOM where global tradeoffs are successfully visualized. SOM's colored by objective functions, aspect ratios and taper ratios reveal various tradeoffs among them.

Furthermore, from the clusters obtained from the SOM, the design variables are mapped onto another SOM. This leads to clusters of design variables which indicate the relative importance of design variables and their interactions. The resulting SOM approximately makes clusters of planform, camber, and others variables without aerodynamic knowledge. This also indicates that SOM is a versatile datamining tool for aeronautical engineering.

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