AEODYNAMIC OPTIMIZATION OF SUPERSONIC TRANSPORT WING USING UNSTRUCTURED ADJOINT METHOD

Hyoung-Jin Kim[1], Daisuke Sasaki[1], Shigeru Obayashi[1], and Kazuhiro Nakahashi[1]

Tohoku University, Sendai, 980-8579 JAPAN

1 Introduction

In the application of gradient-based methods to practical aerodynamic design problems, one of the major concerns is an accurate and efficient calculation of sensitivity derivatives of an aerodynamic objective function. Sensitivity derivatives can be evaluated robustly and efficiently by using a sensitivity analysis code based either on a direct method or on an adjoint method. An adjoint method is preferable in aerodynamic designs because it is more economical when the number of design variables is larger than the total number of an objective function and constraints.

For complex aerodynamic configurations, the unstructured grid approach has several advantages over the structured grid approach. This approach can treat complex geometry with greater efficiency and less effort. It also has a greater flexibility in the adaptive grid refinement/unrefinement; thus the total number of grid points can be saved. Previous works on structured/unstructured sensitivity analysis methods can be found in references. [1-4]

In this study, direct and adjoint sensitivity codes have been developed from a 3-D unstructured Euler solver based on a cell-vertex finite volume method. With the resultant adjoint code, designed are supersonic transport (SST) wings with wing-body-nacelle and wing-body configuration. sensitivities of interior nodes are neglected except those for design variables associated with nacelle translation in order to reduce required computational time for the mesh sensitivity calculation.

2 Sensitivity Analysis

2.1 Direct Method

The discrete residual vector of the nonlinear flow equations is null for a converged flow field solution of steady problems, which can be written symbolically as

$$R[Q, X, \beta] = 0, \tag{1}$$

where Q is a flow variable vector, X grid position vector, β vector of design variables. Direct sensitivity equations can be obtained by direct differentiation of the above equation with respect to β as follows.

$$\frac{dR}{d\beta} = \left[\frac{\partial R}{\partial Q}\right] \frac{dQ}{d\beta} + C,\; where\, C = \left[\frac{\partial R}{\partial X}\right] \frac{dX}{d\beta} + \frac{\partial R}{\partial \beta} \tag{2}$$

The vector C does not depend on the $dQ/d\beta$, and thus, is constant throuthout the solution process. In order to find the solution of Eq.(2) iteratively, a pseudo time term is added to obtain an incremental form, and the LU-SGS scheme is used as was for the flow analysis. The total derivative of the objective function F is given as follows.

$$\frac{dF}{d\beta} = \left[\frac{\partial F}{\partial Q}\right]^T \frac{dQ}{d\beta} + \left[\frac{\partial F}{\partial X}\right]^T \frac{dX}{d\beta} + \frac{\partial F}{\partial \beta} \tag{3}$$

2.2 Adjoint Method

One can introduce adjoint variables and combine Eqs. (2) and (3). Coefficients of the flow variable sensitivity vector $dQ/d\beta$ in the combined equation form the following adjoint equation.

$$\left[\frac{\partial R}{\partial Q}\right]^T \Lambda + \frac{\partial F}{\partial Q} = 0 \tag{4}$$

If one finds the adjoint variable vector Λ which satisfies the above adjoint equation, one can obtain the sensitivity derivative of F with respect to β without any information about the flow variable sensitivity vector $dQ/d\beta$. This makes the computational cost for the sensitivity analysis independent of the number of design variables. Equation (3) eventually becomes to the following form,

$$\frac{dF}{d\beta} = \left[\frac{\partial F}{\partial Q}\right]^T \frac{dQ}{d\beta} + \frac{\partial F}{\partial \beta} + \Lambda^T C \tag{5}$$

The adjoint equation (4) is also solved by the LU-SGS method with a pseudo time term added.

3 Design Methodology

3.1 Design Objective

The objective of the present design study is defined as follows to minimize drag while maintaining a specified lift ${C_L}^*$.

$$F = C_D - \frac{\frac{\partial C_D}{\partial \alpha}}{\frac{\partial C_L}{\partial \alpha}} (C_L - {C_L}^*)$$

3.2 Design Parameters and Grid Modification Method

The wing section geometry is modified adding a linear combination of Hicks and Henne shape functions. We used five design sections along an SST wing span and defined 20 Hicks- Henne design variables and one twist angle per a design section resulting in 105 design variables. For the case with an engine nacelle, the height of diverter is also considered as a design parameter in addition to the 105 design variables. With the new geometry of design sections, vertical coordinates of wing surface node points are linearly interpolated.

For the movement of the grid points with the perturbed surface grid, we used the elliptic partial differential equation method [6] for the grid displacement δ x is used. Required computational time to obtain converged solution was same with that of a few iterations of the Euler solver.

3.3 Grid Sensitivity

The elliptic equation method for the interior grid movement is differentiated to be applied to the grid sensitivity calculation for the vector C in Eq.(5) with respect to each geometric design variable. Since this requires almost the same computational cost with the grid movement procedure, the total computational burden would be a substantial amount if the number of design variables becomes large; say, more than one hundred.

Figure 1 compares the derivatives of the objective function obtained with and without the interior grid sensitivity information for the wing-body-nacelle configuration. Derivatives with respect to the design variables show little difference between the two values except for design variables having indices of 21 - 30, which are defined on the lower surface of the second design section and cause the nacelle to be translated vertically. The nacelle inlet and outlet have sharp edges, and a translation of a body with any singularity such as sharp edges requires interior grid sensitivity information.[2]

In this study, interior grid sensitivities for the ten design variables (21-30) of the design case with nacelle are calculated by the elliptic equation method, while for other design variables, only the surface grid sensitivities are defined.

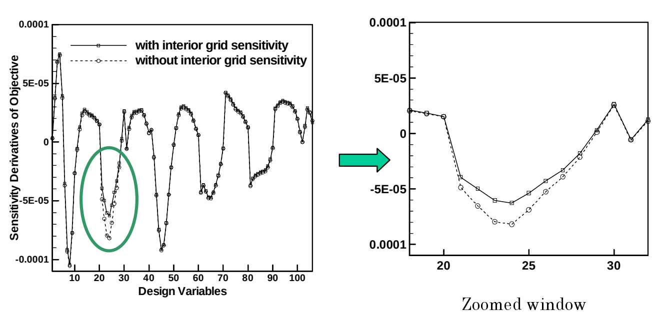

Fig. 1. Comparison of sensitivity derivatives with/without interior grid sens itivity

This simplification approach reduses the computational time for the vector C in Eq.(5) by 75 percent. For the wing-body configuration case, no body-translation is considered, and therefore, interior grid sensitivities for all the design variables can be ignored.

4 Design Results

4.1 Design I; wing-body-nacelle configuration

The present design method is applied to two experimental supersonic transports. One is wing-body-nacelle and the other wing-body configuration, both of which are under development by National Aerospace Laboratory (NAL) of Japan as basic studies for the next generation supersonic transport.[5] Design conditions are a free-stream Mach number of 2.0 and CL of 0.100. The numberof nodes and cells for the adopted volume grid are about 270,000 and 1,500,000, respectively.

As mentioned earlier, total number of design variables is 106. Constraints are imposed so that wing section thickness values at front (5%chord), rear (80%chord) spar position and maximum thickness position (50%chord) should be larger than those of the initial geometry. In addition, the diverter leading edge height is also constrained to be larger than the initial value to prevent boundary layer suction

The SQP optimization [7] was run for five iterations to minimize the objective function with the geometric constraints. However, no further performance improvement was made after three design iterations . As can be seen in Table 1 the drag coefficient was reduced by 16 counts retaining the lift coefficient as the specified value and satisfying imposed thickness constraints by the design.

Figure 2 shows the surface pressure contours on the wing lower surface. It can be noted that the strength of the impinging shock wave on the wing lower surface generated by the diverter leading edge is greatly reduced through the design procedure.

The leading-edge height of the diverter remained the same as the initial value, since the gradient of the objective function with respect to the height is

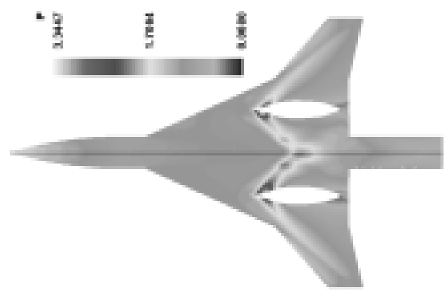

Fig. 2. Lower surface pressure contour of NAL experimental supersonic transport with nacelles, lower:initial, upper: design

positive throughout the design iterations. This is quite natural in a sense that the diverter height increment will increase the aircraft volume and also the pressure drag accordingly.

4.2 Design II; wing-body configuration

Design conditions are the same as previous; free-stream Mach number of 2.0 and CL of 0.100. The initial wing geometry has been designed by an inverse design method with the natural laminar flow (NLF) concept, and shows very good aerodynamic performance at the design condition.[5] The number of nodes and cells for the adopted volume grid are about 260,000 and 1,390,000, respectively. In the present optimization, the same design variables are employed as the design example I except the diverter leading edge height. Therefore, totally 105 design variables are used for the wing section shape modification and twist angles variation. The same thickness constraints are also imposed as example I.

Table 1 summarizes the design results. The SQP optimizer was run for fifteen iterations to obtain a drag coefficient reduced by only one count from 0.006349 to 0.006242 retaining the lift coefficient as the specified value and satisfying imposed thickness constraints.

Figure 3 compares wing section shapes and pressure distributions at a wing section. Pressure distributions show that the suction peak at the leading edge has been increased by the design with other features of pressure distributions being almost the same although the section shapes have been changed remarkably. This implies that the initial shape is already near an optimum, and performance improvement is hard to be made from the initial one. The increased suction peaks of the design wing, therefore, does not meet the NLF concept employed as a design philosophy for the initial wing. Instead, the present design utilizes the leading-edge suction by taking the advantage of the inviscid computation.

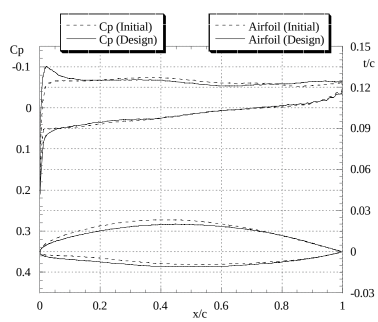

Fig. 3. Wing section shapes and pressure distributions for design example II ($\eta = 0.3$)

Table 1. Design results for SST wing-body-nacelle configuration

	example I	(wing-body-nacelle)		example II	(wing-body)	
	Initial	Design	Δ(%)	Initial	Design	Δ(%)
C_L	0.10017	0.10020	+0.03	0.10002	0.09993	-0.088
C_D	0.020513	0.018918	+7.78	0.006349	0.006242	1.675
L/D	4.883	5.297	+8.48	15.75	16.01	1.614

5 Conclusion Remarks

An aerodynamic design optimization system is developed using the unstructured Euler solver and the discrete adjoint method. For an efficient calculation of terms related with the grid sensitivities, grid sensitivities of interior node points are ignored except those for the design variables associated with nacelle translation. The present method is successfully applied to design the SST wing-body-nacelle and wing-body configurations. For the wing-body-nacelle configuration, the impinging shock wave from the diverter on the wing lower surface has been greatly reduced by five iterations of the SQP optimizer. On the other hand, the initial shape of the wing-body configuration was near optimum itself, and only one count drag reduction was made by fifteen design iterations.

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