

# MULTIPOINT AERODYNAMIC DESIGN OF SUPERSONIC WING PLANFORM USING MOGA

**Shigeru Obayashi and Yukihiro Takeguchi**

*Department of Aeronautics and Space Engineering, Tohoku University*

Aoba-yama 01, Sendai, 980-8579

Phone: +81-22-217-6980, Fax: +81-22-217-6979

s.obayashi@computer.org (<http://www.ad.mech.tohoku.ac.jp/~obayashi>)

## ABSTRACT

This paper describes the design optimization of a planform shape for a supersonic transport wing using Multiple Objective Genetic Algorithm. The objective functions are to minimize the drag for supersonic cruise, the drag for transonic cruise and the bending moment at the wing root for supersonic cruise. The planform shape is defined by six design variables. An Euler flow code is used to evaluate the supersonic drag, and a potential flow code is used to evaluate the transonic drag. To reduce the total computational time, flow calculations were parallelized on NEC SX-4 computer using 32 PE's. Physically reasonable Pareto solutions were obtained from the present optimization. One of the present Pareto solutions is found to outperform the existing wing design in all design objectives.

## INTRODUCTION

Commercial aviation has grown remarkably with the development of the world economy during a half-century. This growth of air traffic is projected to continue well into the 21<sup>st</sup> century with increase demands for more efficient aircraft. As a solution to such demands, the next generation supersonic transport has been considered worldwide [1].

Concorde is the only commercially operating, first generation supersonic transport. Although it can fly at speed of Mach 2.0, most of airline companies did not purchase it for their services. There are three major deficiencies regarding Concorde. One is its relatively high operating cost. Second is the community noise at the airport for taking off and landing. The third is the sonic boom at the supersonic cruise. The first two deficiencies are ascribed to its low lift-to-drag ratio. To generate enough lift necessary for the flight, Concorde has to burn more fuel and thus make more noise. To improve the lift-to-drag ratio of the next generation supersonic transport, aerodynamic optimization of the aircraft configuration has been investigated.

The third problem, however, has a fundamental difficulty since sonic-boom minimization is in conflict

with the drag minimization. On the other hand, acceptability of supersonic transport is very sensitive to the sonic boom over populated areas. Thus, one of the design choices is to limit supersonic flight over sea and to have transonic flight over land. Although such decision excludes the sonic boom from the design consideration, the design is now faced with transonic performance of the aircraft.

This paper considers multipoint aerodynamic optimization of a wing planform shape for supersonic aircraft both at supersonic cruise condition and at transonic cruise condition. Aerodynamic drag will be minimized at both cruise conditions under lift constraints. Aerodynamic optimization of the wing planform, however, drives the wing to have an impracticably large aspect ratio. Therefore, minimization of the wing root bending moment is added as the third design objective.

The present multipoint design problem can be regarded as multiobjective (MO) optimization. Solutions to MO problems are often computed by combining multiple criteria into a single criterion according to some utility function. In many cases, however, the utility function is not well known prior to the optimization process. The whole problem should then be treated with non-commensurable objectives. MO optimization seeks to optimize the components of a vector-valued objective function. Unlike single objective optimization, the solution to this problem is not a single point, but a family of points known as the Pareto-optimal set.

By maintaining a population of solutions, Genetic Algorithms (GAs) can search for many Pareto-optimal solutions in parallel. This characteristic makes GAs very attractive for solving MO problems. As a solver for MO problems, the following two features are desired: 1) the solutions obtained are Pareto-optimal and 2) they are uniformly sampled from the Pareto-optimal set. To achieve these, MOGAs have successfully been introduced by Fonseca and Fleming [2].

Furthermore, it was shown that the so-called best- $N$  selection helps to find the extreme Pareto solutions [3]. The best- $N$  selection picks up the best  $N$  individuals

among  $N$  parents and  $N$  children for the next generation similar to CHC [4]. The extreme Pareto solutions are the optimal solutions of the single objectives. By examining the extreme Pareto solutions, quality of Pareto solutions can be measured. The present MO problem will be solved by using MOGA coupled with the best- $N$  selection.

### APPROACH

In GAs, the natural parameter set of the optimization problem is coded as a finite-length string. Traditionally, GAs use binary numbers to represent such strings: a string has a finite length and each bit of a string can be either 0 or 1. For real function optimization, however, it is more natural to use real numbers. The length of the real-number string corresponds to the number of design variables.

#### Crossover and Mutation

A simple crossover operator for real number strings is the average crossover [5] which computes the arithmetic average of two real numbers provided by the mated pair. In this paper, a weighted average is used as

$$\begin{aligned} \text{Child1} &= \text{ran1} \cdot \text{Parent1} + (1-\text{ran1}) \cdot \text{Parent2} \\ \text{Child2} &= (1-\text{ran1}) \cdot \text{Parent1} + \text{ran1} \cdot \text{Parent2} \end{aligned} \quad (1)$$

where Child1,2 and Parent1,2 denote encoded design variables of the children (members of the new population) and parents (a mated pair of the old generation), respectively. The uniform random number  $\text{ran1}$  in  $[0,1]$  is regenerated for every design variable.

Mutation takes place at a probability of 20% (when a random number satisfies  $\text{ran2} < 0.2$ ) initially and the rate is going to decline linearly during the evolution. Equations (1) will then be replaced by

$$\begin{aligned} \text{Child1} &= \text{ran1} \cdot \text{Parent1} + (1-\text{ran1}) \cdot \text{Parent2} + m \cdot (\text{ran3} - 0.5) \\ \text{Child2} &= (1-\text{ran1}) \cdot \text{Parent1} + \text{ran1} \cdot \text{Parent2} + m \cdot (\text{ran3} - 0.5) \end{aligned} \quad (2)$$

where  $\text{ran2}$  and  $\text{ran3}$  are also uniform random numbers in  $[0,1]$  and  $m$  determines the range of possible mutation.

#### Multiobjective Pareto Ranking

To search Pareto-optimal solutions by using MOGA, the ranking selection method [6] can be extended to identify the near-Pareto-optimal set within the population of GA. To do this, the following definitions are used: suppose

$\mathbf{x}_i$  and  $\mathbf{x}_j$  are in the current population and  $\mathbf{f} = (f_1, f_2, \dots, f_q)$  is the set of objective functions to be maximized,

- $\mathbf{x}_i$  is said to be dominated by (or inferior to)  $\mathbf{x}_j$ , if  $\mathbf{f}(\mathbf{x}_i)$  is partially less than  $\mathbf{f}(\mathbf{x}_j)$ , i.e.,  $f_1(\mathbf{x}_i) \leq f_1(\mathbf{x}_j) \wedge f_2(\mathbf{x}_i) \leq f_2(\mathbf{x}_j) \wedge \dots \wedge f_q(\mathbf{x}_i) \leq f_q(\mathbf{x}_j)$  and  $\mathbf{f}(\mathbf{x}_i) \neq \mathbf{f}(\mathbf{x}_j)$ .
- $\mathbf{x}_i$  is said to be non-dominated if there doesn't exist any  $\mathbf{x}_j$  in the population that dominates  $\mathbf{x}_i$ .

Non-dominated solutions within the feasible region in the objective function space give the Pareto-optimal set.

Let's consider the following optimization:

$$\begin{aligned} \text{Maximize:} & \quad f_1 = x, \quad f_2 = y \\ \text{Subject to:} & \quad x^2 + y^2 \leq 1 \quad \text{and} \quad 0 \leq x, y \leq 1 \end{aligned}$$

The Pareto front of the present test case becomes a quarter arc of the circle  $x^2 + y^2 = 1$  at  $0 \leq x, y \leq 1$ . Consider an individual  $\mathbf{x}_i = (x, y)_i$  at generation  $t$  (Fig. 1) which is dominated by pit individuals in the current population. Following [2], its current position in the individuals' rank can be given by

$$\text{rank}(\mathbf{x}_i, t) = 1 + p_i^t \quad (3)$$

All non-dominated individuals are assigned rank 1 as shown in Fig. 1. The fitness values are reassigned according to rank as an inverse of their rank values. Then the SUS method [7] takes over with the reassigned values.

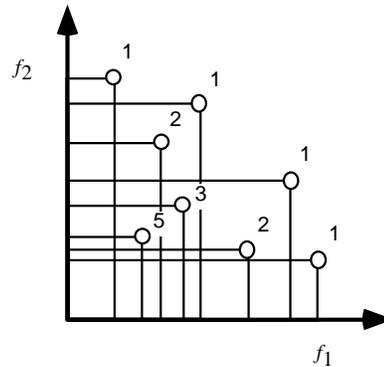


Fig. 1. Pareto ranking method.

## Fitness Sharing

To sample Pareto-optimal solutions from the Pareto-optimal set uniformly, it is important to maintain genetic diversity. It is known that the genetic diversity of the population can be lost due to the stochastic selection process. This phenomenon is called the random genetic drift. To avoid such phenomena, the niching method has been introduced [6].

The model used here is called fitness sharing (FS). A typical sharing function is given by Goldberg [6]. The sharing function depends on the distance between individuals. The distance can be measured with respect to a metric in either genotypic or phenotypic space. A genotypic sharing measures the interchromosomal Hamming distance. A phenotypic sharing can further be classified into two types. One measures the distance between the decoded design variables. The other, on the other hand, measures the distance between the designs' objective function values. Here, the latter phenotypic sharing is employed since we seek a global tradeoff surface in the objective function space.

This scheme introduces new GA parameters, the niche size  $\sigma_{share}$ . The choice of  $\sigma_{share}$  has a significant impact on the performance of MOGAs. Fonseca and Fleming [3] gave a simple estimation of  $\sigma_{share}$  in the objective function space as

$$N\sigma_{share}^{q-1} - \frac{\prod_{i=1}^q (M_i - m_i + \sigma_{share}) - \prod_{i=1}^q (M_i - m_i)}{\sigma_{share}} = 0 \quad (4)$$

where  $N$  is a population size,  $q$  is a dimension of the objective vector, and  $M_i$  and  $m_i$  are maximum and minimum values of each objective, respectively. This formula has been successfully adapted here. Since this formula is applied at every generation, the resulting  $\sigma_{share}$  is adaptive to the population during the evolution process. Niche counts can be consistently incorporated into the fitness assignment according to rank by using them to scale individual fitness within each rank.

## Multipoint Design Objectives

Flow conditions and lift coefficients considered here are  $M_\infty = 2.0$  and  $C_L = 0.1$  for supersonic cruise and  $M_\infty = 0.9$  and  $C_L = 0.15$  for transonic cruise. The primary objective of the present research is to minimize drag coefficients both at the supersonic cruise and at the transonic cruise.

To determine the aerodynamic drag, the entire flow field has to be solved. The supersonic inviscid drag is evaluated by using an Euler flow solver [8]. The

transonic inviscid drag is evaluated by using a full potential flow solver [9]. The lift constraints were satisfied by adjusting angles of attack. These flow analyses dominate most of the computational time required for the optimization. Thus, a simple master-slave parallelization is employed: MOGA operators are assigned to a master processor and the flow evaluations are distributed to slave processors. Calculations were performed on NEC SX-4 computer at Computer Center of Tohoku University, using 32 PE's (this corresponds to one node of SX-4, a quarter of the center machine, and the node's peak performance is 64 GFLOPS with 8 GB memory). The population size was thus set to 64.

The preliminary optimization of the present multipoint design, however, revealed that all Pareto solutions have impracticably large aspect ratio. This was typical when the structural constraint was not considered. To account for the structural integrity, the third objective is introduced to reduce the bending moment at the wing root. The bending moment is evaluated by directly integrating the pressure load at the supersonic cruise condition.

## RESULTS

Design variables are illustrated in Fig. 2 and their ranges are listed in Table 1. In total of six design variables are used. A wing area is fixed as required for takeoff and landing performance. A chord length at the wing tip is automatically determined due to the given wing area. An airfoil shape is also frozen to NACA64A0003. Neither camber nor twist is considered.

Figure 3 shows the resulting Pareto solutions in the three dimensional objective function space as well as their two-dimensional projections. The evolution was stopped after 35 generations since the Pareto front became almost steady. The total computational time was roughly 46 hours. Tradeoffs between the objectives can be identified more easily in the two-dimensional projections. For example, a Pareto front can be found in the two-dimensional projection of supersonic versus transonic drag coefficients. Our preliminary calculation for minimizing the supersonic and transonic drag coefficients resulted in a similar Pareto front. However, all the planform shapes corresponding to this Pareto front appeared to have impracticably large aspect ratios. Therefore the bending moment was introduced as the third design objective. As shown in the other two-dimensional projections, the minimization of the bending moment has tradeoffs with minimization of the aerodynamic drags.

Figure 4 illustrates several planform shapes among

the Pareto solutions. Aerodynamically good solutions have very large aspect ratios. In an inviscid flow, aerodynamic drag has two components: the wave drag and the induced drag. Since the induced drag is inversely proportional to the aspect ratio of a wing, an aerodynamically optimized wing should have a large aspect ratio. On the other hand, the wave drag primarily relates to the aircraft speed and the leading-edge sweep angle. As the aircraft flies faster, the sweep angle should be increased. As shown in the figure, the extreme Pareto solution corresponding to the minimum supersonic drag has a much larger sweep angle than the one corresponding to the minimum transonic drag. Finally, the solution corresponding to the minimum bending moment gives a very small aspect ratio. Since the amount of lift is constrained, a smaller aspect ratio simply gives a smaller bending moment. These extreme Pareto solutions appear physically reasonable.

To examine the present optimization result further, several Pareto solutions are compared with the existing planform shapes of Concorde and National Aerospace Laboratory's scaled supersonic experimental airplane [10] shown in Fig. 5. NAL has developed a series of aerodynamic designs for their experimental airplane and this particular planform shape corresponds to their second design. For the comparison purpose, only the original planform shapes of those wings were used while the airfoil shapes were changed to NACA64A003. In addition, these planform shapes were extended to the center line of the body. Although Concorde's planform is beyond the present design range, it is expected to give a reference for the supersonic drag.

Table 2 summarizes the comparison of their performances. The present solutions are confirmed to have the best values in the transonic and supersonic drag coefficients, respectively. For the bending moment, Concorde's planform has the smallest value since it has a much smaller aspect ratio due to a larger leading-edge sweep angle than the present solutions. In addition, Concorde's planform cannot produce the lift constrained for the transonic cruise. Since FLO27 was diverged by increasing the angle of attack, the transonic drag was listed as diverged. NAL's design was basically optimized for the supersonic cruise, but its aspect ratio was enlarged more than Concorde's to improve the low speed performance. This design is reconfirmed to perform reasonably well in all three objectives. One of the present Pareto solutions is found to have the best performance in all three objectives. This shape is referred as sample Pareto solution in Fig. 4. It has the largest improvement in the transonic drag compared with NAL's design.

## CONCLUSION

The multipoint design optimization of planform shapes for a supersonic transport wing has been performed by using MOGA. The objective functions are to minimize the drags at supersonic and transonic cruise conditions. To account for the structural constraint, minimization of the bending moment at the wing root for supersonic cruise is considered as the third design objective. The wing planform shape is defined by six design variables.

Physically reasonable Pareto solutions were obtained from the present optimization. The results identify that the aspect ratio is one of the important factors for the aerodynamic performance. Although aerodynamically good solutions tend to have impractically large aspect ratios, compromised solutions with the bending moment give good candidates for a practical design. One of the present Pareto solutions is found to outperform the existing wing design in all design objectives. In future, the present approach should be extended to optimize the twist, camber and thickness distributions of the supersonic wing.

## ACKNOWLEDGEMENT

This research was funded by Japanese Government's Grants-in-Aid for Scientific Research, No. 10305071. The first author's research has been partly supported by Bombardier Aerospace, Toronto. The authors would like to thank National Aerospace Laboratory's SST Design Team for providing many useful data.

## REFERENCES

1. Sobieczky, S. (ed.) 1997. *New Design Concepts for High Speed Air Transport*, CISM Courses and Lectures No. 366, Wien New York, Springer.
2. Fonseca, C. M. and Fleming, P. J. 1993. Genetic algorithms for multiobjective optimization: formulation, discussion and generalization. In *Proceedings of the 5th International Conference on Genetic Algorithms*, 416-423. San Mateo, Calif.: Morgan Kaufmann Publishers.
3. Obayashi, S., Takahashi, S. and Takeguchi, Y. 1998. Niching and Elitist Models for MOGAs. In *Parallel Problem Solving from Nature - PPSN V*, 260-269, Lecture Notes in Computer Science. Berlin, Germany: Springer.
4. Eshelman, L. J. 1991. The CHC Adaptive Search Algorithm: How to Have Safe Search When Engaging in Nontraditional Genetic Recombination.

In Foundations of Genetic Algorithms, 265-283. San Mateo, Calif.: Morgan Kaufmann Publishers.

5. Davis, L. 1990. Handbook of Genetic Algorithms, Reinhold, New York: Van Nostrand.
6. Goldberg, D. E. 1989. Genetic Algorithms in Search, Optimization & Machine Learning. Reading, Mass.: Addison-Wesley.
7. Baker, J. E. 1987. Reducing bias and inefficiency in the selection algorithm. In Proceedings of the Second International Conference on Genetic Algorithms, 14-21. San Mateo, Calif.: Morgan Kaufmann Publishers.
8. Obayashi, S., Nakahashi, K., Oyama, A. and Yoshino, N. 1998. Design Optimization of Supersonic Wings Using Evolutionary Algorithms. In Proceedings of the Fourth ECCOMAS Computational Fluid Dynamics Conference, Vol. 2, 575-579. Chichester, UK: John Wiley & Sons.
9. Jameson, A. and Caughey, D. A. 1977. A Finite Volume Method For Transonic Potential Flow Calculations. AIAA paper 77-677, AIAA.
10. Iwamiya, T. 1998. NAL SST project and Aerodynamic Design of Experimental aircraft, In Proceedings of the Fourth ECCOMAS Computational Fluid Dynamics Conference, Vol. 2, 580-585. Chichester, UK: John Wiley & Sons.

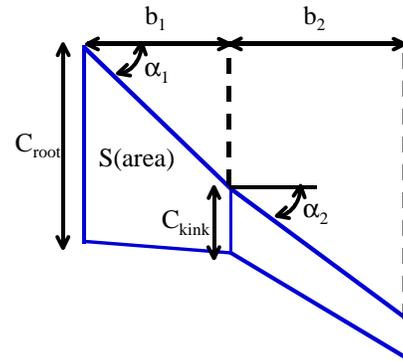


Fig. 2 Planform definition.

Variable	Range
$\alpha_1$	35~70(deg)
$\alpha_2$	35~70(deg)
$b_1$	2~7
$b_2$	2~7
$C_{root}$	10~20
$C_{kink}$	3~15

Table 1 Ranges of design variables.

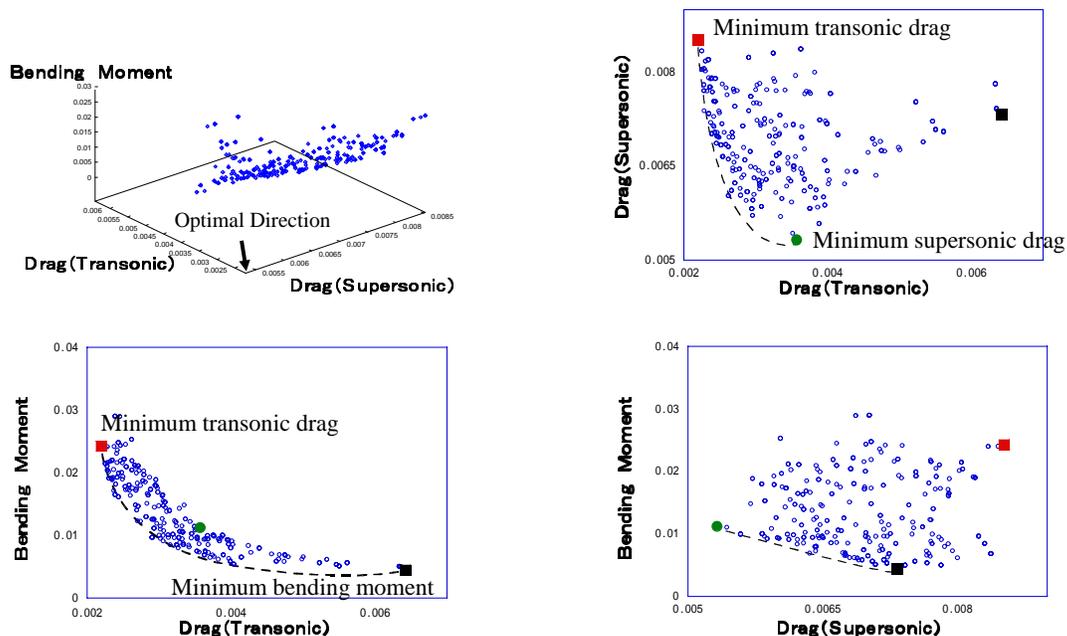


Fig. 3 Pareto solutions in the objective function space.

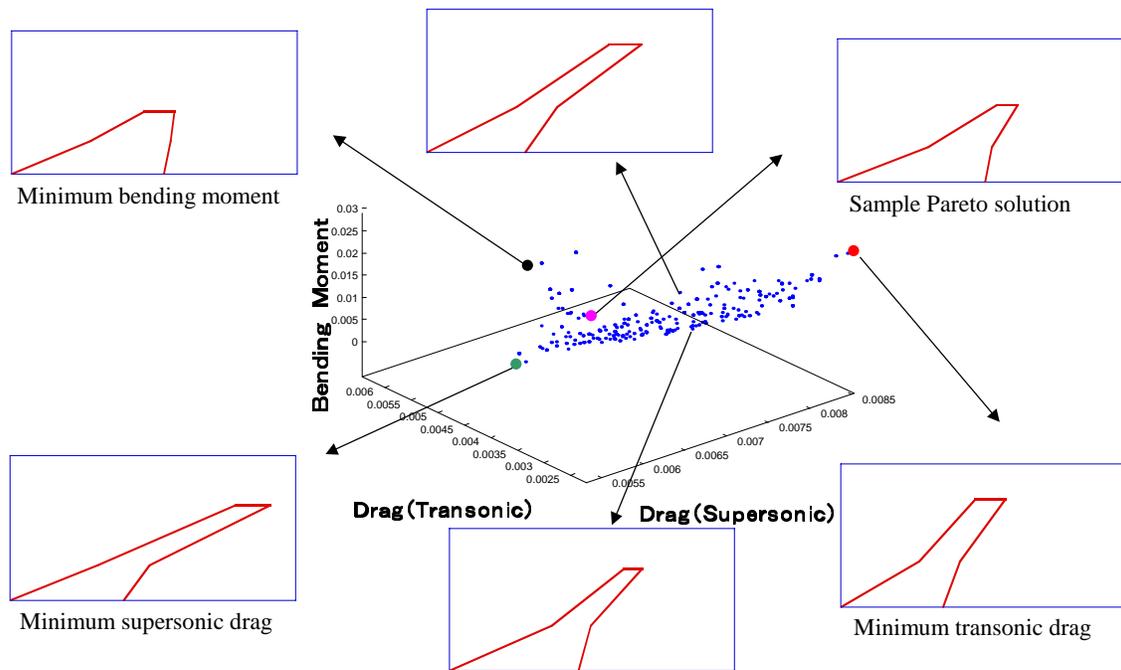


Fig. 4 Planform shapes of Pareto solutions.

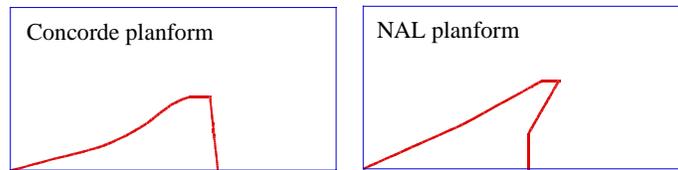


Fig. 5 Existing planform shapes.

	Transonic drag ( $\times 10^{-4}$ )	Supersonic drag ( $\times 10^{-4}$ )	Bending moment ( $\times 10^{-4}$ )
Concorde	(diverged)	<b>75.62</b>	<b>25.86</b>
NAL 2nd	<b>49.88</b>	<b>70.07</b>	<b>60.87</b>
Minimum transonic drag	<b>21.99</b>	<b>85.21</b>	<b>242.5</b>
Minimum supersonic drag	<b>35.73</b>	<b>53.24</b>	<b>112.2</b>
Minimum bending moment	<b>64.24</b>	<b>73.29</b>	<b>44.07</b>
Sample Pareto solution	<b>46.33</b>	<b>69.39</b>	<b>58.78</b>

Table 2 Performance comparison.