Multidisciplinary Design Optimization of Aircraft Wing Planform Based on Evolutionary Algorithms

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ABSTRACT

This paper examines the evolutionary approach for aircraft design optimization. Several niching and elitist models are first applied to Multiple-Objective Genetic Algorithms (MOGAs). Numerical results suggest that the combination of the fitness sharing and the best-*N* selection leads to the best performance. The resulting MOGA is then applied to multidisciplinary design optimization problems of transonic and supersonic wing planform shapes. The results confirm the feasibility of the present approach.

1. INTRODUCTION

Aircraft design presents a grand challenge to numerical optimization. It is in nature multidisciplinary among aerodynamics, structure, control and propulsion. Each disciplinary model has to be accurate enough to predict aircraft performance. Especially, aerodynamic calculation is computer intensive and the resulting aerodynamic performance is very sensitive to the geometry. Therefore, a robust optimization algorithm is indispensable to this field.

Evolutionary algorithms, Genetic Algorithms (GAs) for example, are known to be robust [1] and have been enjoying increasing popularity in the field of numerical optimization in recent years. GAs have been applied to aerodynamic optimization using Computational Fluid Dynamics (CFD) (for example, see [2]).

Furthermore, GAs can search for many Pareto-optimal solutions in parallel, by maintaining a population of solutions [1]. Most real world problems require the simultaneous optimization of multiple, often competing objectives. Such multiobjective (MO) problems seek to optimize components of a vector valued objective function. Unlike the single-objective optimization, the solution to MO problem is not a single point, but a family of points known as the Pareto-optimal set. Each point in this set is optimal in the sense that no improvement can be achieved in one objective component that doesn't lead to degradation in at least one of the remaining components.

GAs can be very efficient, if they can sample solutions uniformly from the Pareto-optimal set. Since GAs are inherently robust, the combination of efficiency and robustness makes them very attractive for solving MO problems. Several approaches have been proposed [3-5] and one of them to be employed here is called Multiple Objective Genetic Algorithms (MOGAs) [4].

Performance of MOGAs can be measured by variety of Pareto solutions and convergence to the Pareto front. To construct a better MOGA, several niching and elitist models are examined in this paper through numerical tests. The resulting GA will be applied to multidisciplinary design optimization (MDO) of aircraft planform shapes.

2. MOGAS

The first three sections below describe basic GA operators used here. Then the extension to MO problems are discussed. Finally, the niching and elitist models are introduced.

Coding

In GAs, the natural parameter set of the optimization problem is encoded as a finite-length string. Traditionally, GAs use binary numbers to represent such strings: a string has a finite length and each bit of a string can be either 0 or 1. For real function optimization, it is more natural to use a vector representation of real numbers. The length of the real-number string corresponds to the number of design variables.

As a sample test case, let's consider the following optimization:

$$\text{Maximize:} \quad f(x, y) = x + y$$
$$\text{Subject to:} \quad x^2 + y^2 \leq 1 \text{ and } 0 \leq x, y \leq 1$$

Each point (x, y) in the GA population is encoded by a string (r, θ) in the polar coordinates since the representation of the constraints will be simplified..

Crossover and Mutation

A simple crossover operator for real number strings is the average crossover which computes the arithmetic average of two real numbers provided by the mated pair. In this paper, a weighted average is used as

$$\begin{aligned} \text{Child1} &= ran1 \cdot \text{Parent1} + (1 - ran1) \cdot \text{Parent2} \\ \text{Child2} &= (1 - ran1) \cdot \text{Parent1} + ran1 \cdot \text{Parent2} \end{aligned} \qquad (1)$$

where Child1,2 and Parent1,2 denote encoded design variables of the children (members of the new population) and parents (a mated pair of the old generation), respectively. The uniform random number *ran*1 in [0,1] is regenerated for every design variable. Because of Eqs. (1), the number of the initial population is assumed even.

Mutation takes place at a probability of 20% (when a random number satisfies $ran2 < 0.2$). Equations (1) will then be replaced by

$$\text{Child1} = ran1 \cdot \text{Parent1} + (1\text{-}ran1) \cdot \text{Parent2} + m \cdot (ran2\text{-}0.5)$$
$$\text{Child2} = (1\text{-}ran1) \cdot \text{Parent1} + ran1 \cdot \text{Parent2} + m \cdot (ran3\text{-}0.5) \quad (2)$$

where *ran*2 and *ran*3 are also uniform random numbers in [0,1] and *m* determines the range of possible mutation. In the following test cases, *m* was set to 0.4 for the radial coordinate r and $\pi/3$ for the angular coordinate θ.

Ranking

For a successful evolution, it is necessary to keep appropriate levels of selection pressure throughout a simulation [1]. Scaling of objective function values has been used widely in practice. However, this leaves the scaling procedures to be determined. To avoid such parametric procedures, a ranking method is often used. In this method, the population is sorted according to objective function value. Individuals are then assigned an offspring count that is solely a function of their rank. The best individual receives rank 1, the second best receives 2, and so on. The fitness values are reassigned according to rank, for example, as an inverse of their rank values. Then the usual stochastic universal sampling method takes over with the reassigned values. The method described so far will be hereon referred to as SOGA (Single-Objective Genetic Algorithm).

Pareto Ranking for MO Problems

SOGA assumes that the optimization problem has (or can be reduced to) a single criterion (or objective). Most engineering problems, however, require the simultaneous optimization of multiple, often competing criteria. Solutions to MO problems are often computed by combining multiple criteria into a single criterion according to some utility function. In many cases, however, the utility function is not well known prior to the optimization process. The whole problem should then be treated with non-commensurable objectives. MO optimization seeks to optimize the components of a vector-valued objective function. Unlike single objective optimization, the solution to this problem is not a single point, but a family of points known as the Pareto-optimal set.

By maintaining a population of solutions, GAs can search for many Pareto-optimal solutions in parallel. This characteristic makes GAs very attractive for solving MO problems. As solvers for MO problems, the following two features are desired: 1) the solutions obtained are Pareto-optimal and 2) they are uniformly sampled from the Pareto-optimal set. To achieve these with GAs, the Pareto-ranking and fitness sharing techniques were successfully combined into MOGAs [4].

To search Pareto-optimal solutions by using MOGA, the ranking selection method described above for SOGA can be extended to identify the near-Pareto-optimal set within the population of GA. To do this, the following definitions are used: suppose $\mathbf{x}_i$ and $\mathbf{x}_j$ are in the current population and $\mathbf{f} = (f_1, f_2)$ is the set of objective functions to be maximized,

1. $\mathbf{x}_i$ is said to be dominated by (or inferior to) $\mathbf{x}_j$, if $\mathbf{f}(\mathbf{x}_i)$ is less than $\mathbf{f}(\mathbf{x}_j)$, i.e., $f_1(\mathbf{x}_i) \le f_1(\mathbf{x}_j) \wedge f_2(\mathbf{x}_i) \le f_2(\mathbf{x}_j)$ and $\mathbf{f}(\mathbf{x}_i) \ne \mathbf{f}(\mathbf{x}_j)$.
2. $\mathbf{x}_i$ is said to be non-dominated if there doesn't exist any $\mathbf{x}_j$ in the population that dominates $\mathbf{x}_i$.

Non-dominated solutions within the feasible region in the objective function space give the Pareto-optimal set.

As the first test case in this paper, let's consider the following optimization:

Maximize: $f_1 = x, \quad f_2 = y$

Subject to: $x^2 + y^2 = 1$ and $0 \le x, y \le 1$

The Pareto front of the present test case becomes a quarter arc of the circle $x^2 + y^2 = 1$ at $0 \le x, y \le 1$.

Consider an individual $\mathbf{x}_i$ at generation t (Fig. 1) which is dominated by p_i^t individuals in the current population. Its current position in the individuals' rank can be given by

$$\text{rank}(\mathbf{x}_i, t) = 1 + p_i^t \quad (3)$$

All non-dominated individuals are assigned rank 1 as shown in Fig. 1. The fitness assignment according to rank can be done similar to that in SOGA.

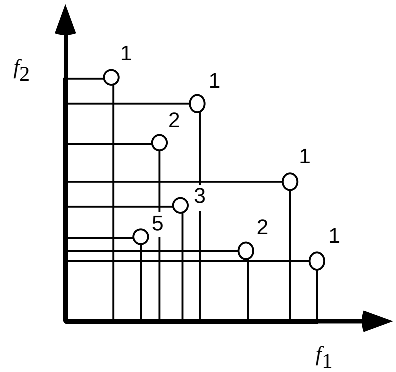

Fig. 1. Pareto ranking method.

Fitness Sharing

To sample Pareto-optimal solutions from the Pareto-optimal set uniformly, it is important to maintain genetic diversity. It is known that the genetic diversity of the population can be lost due to their stochastic selection process. This phenomenon is called the random genetic drift. To avoid such phenomena, the niching method has been introduced [1]. In this paper, two specific niching models are examined for MOGAs.

The first model is called the fitness sharing (FS). A typical sharing function is given by Goldberg [1]. The sharing function depends on the distance between individuals. The distance can be measured with respect to a metric in either genotype or phenotypic space. A genotype sharing measures the interchromosomal Hamming distance. A phenotypic sharing, on the other hand, measures the distance between the designs' objective function values. In MOGAs, a phenotypic sharing is usually preferred since we seek a global tradeoff surface in the objective function space.

This scheme introduces a new GA parameters, the niche size σ_{share}. The choice of σ_{share} has a significant impact on the performance of MOGAs. Fonseca et al. [4] gave a simple estimation of σ_{share} in the objective function space. It has been successfully adapted here. Since this formula is applied at every generation, the resulting σ_{share} is adaptive during the evolution process. Niche counts can be consistently incorporated into the fitness assignment according to rank by using them to scale individual fitness within each rank.

Coevolutionary Shared Niching

Coevolutionary shared niching (CSN) is an alternate, new niching method proposed in Goldberg et al. [6]. The technique is loosely inspired by the economic model of monopolistic competition, in which businessmen locate themselves among geographically distributed populations – businessmen and customers – where individuals in each population seek to maximize their separate interests thereby creating appropriately spaced niches containing the most highly fit individuals. The customer population may be viewed as a modification to the original sharing scheme, in which the sharing function and σ_{share} are replaced by requiring customers to share within the closest businessman's service area. The evolution of the businessman population is conducted in a way that promotes the independent establishment of the most highly fit regions or niches in the search space. The businessman population is created by an *imprint* operator that carries the best of one population over the other. Simply stated, businessmen are chosen from the best of the customer population.

This model introduces a new GA parameter d_{min} that determines the minimum distance between the businessmen. In the following test cases, this parameter d_{min} was tuned by the try-and-error basis and kept constant during the evolution. Niche counts was incorporated into the fitness assignment according to rank similar to the fitness sharing.

Elitist Models

To examine effects of generational models, three models are considered here. The first one is the simple generational (SG) model that replaces N parents simply with N children. The second one is the elitist recombination (ER) model that selects two best individuals among two parents and their two offsprings. The final model is the so-called best-N (BN) model that selects the best N individuals among N parents and N children similar to CHC [7]. The population size was kept to 100 in all test cases.

3. COMPARISON OF NICHING AND ELITIST MODELS

From the techniques described above, five optimization results are shown here for the first test case. Figures 2 to 4 show the results obtained from the simple generational model with the fitness sharing (SG + FS), the elitist recombination with the fitness sharing (ER + FS) and the best-N with the fitness sharing (BN + FS), respectively. The GA population is represented by dots and the Pareto front is indicated by a solid arc. When FS was used, the results were improved by stronger elitist models. Among the three models examined here, the best-N selection BN was the best elitist model.

Figure 5 shows the result obtained from SG + CSN. It shows that the coevolutionary shared niching CSN provides a significant improvement over FS. However, when CSN is combined with BN as shown in Fig. 6, the result is slightly worse than that by BN + FS.

Note that the present FS uses the adaptive σ_{share}. From the observation, the performance of the niching models can be summarized as

$$\text{constant } \sigma_{share} < \text{constant } d_{min} < \text{adaptive } \sigma_{share}$$

This leads to a speculation: "adaptive σ_{share} < adaptive d_{min} ?" CSN is very promising but further investigations will be needed, especially in the area of how to determine its parameter d_{min}.

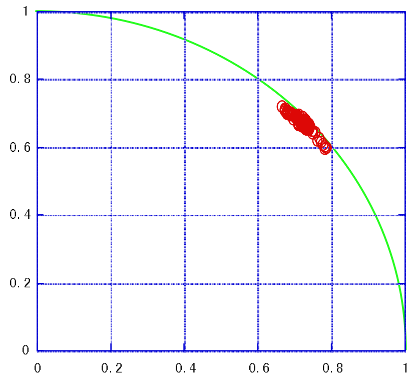

Fig. 2. Pareto solutions obtained from SG + FS.

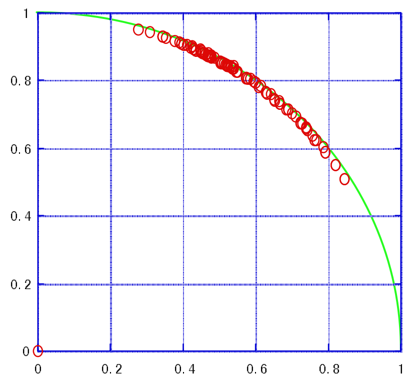

Fig. 3. Pareto solutions obtained from ER + FS.

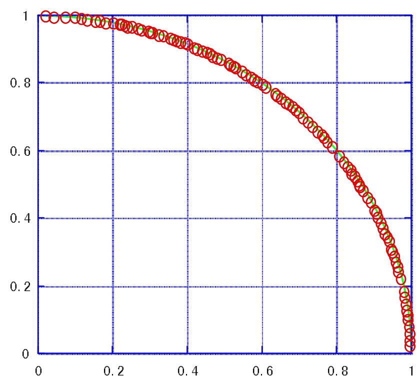

Fig. 4. Pareto solutions obtained from BN + FS.

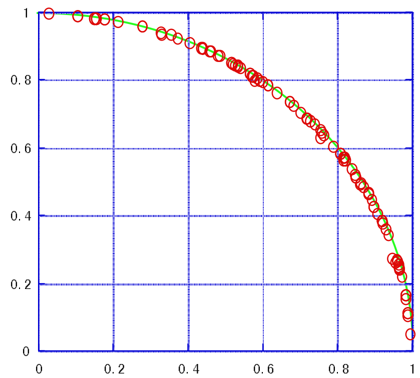

Fig. 5. Pareto solutions obtained from SG + CSN.

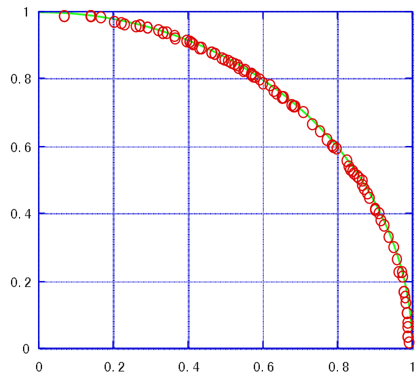

Fig. 6. Pareto solutions obtained from BN + CSN.

4. MULTIDISCIPLINARY OPTIMIZATION OF WING PLANFORM DESIGN

Transonic Wing Planform Design

Aerodynamic optimization often has to account for constraints, for example, structural strength. Such structural constraints might be derived from design optimization in the structural discipline. However, a simple sequential optimization that executes each disciplinary optimization task in sequence cannot take advantage of beneficial cross-disciplinary tradeoffs. Therefore, MDO approach is desired. Formulation of such approach presents organizational challenges for coupling analysis codes in each discipline. Furthermore, MDO requires multiobjective, system-level optimization.

An application of MOGA to MDO of transonic wing planform design [8] is first examined in this section. The present MO optimization problem is described as follows:

1. Minimize aerodynamic drag (induced + wave drag)
2. Minimize wing weight
3. Maximize fuel weight (tank volume) stored in wing

under these constraints:

1. Lift to be greater than given aircraft weight
2. Structural strength to be greater than aerodynamic loads

Since the purpose of the present design is to examine the performance of MOGAs as a system-level optimizer, the number of design variables for wing geometry is greatly reduced. First, aircraft sizes were assumed as wing area of 525 ft^2 total maximum takeoff weight of 45,000 *lb* at cruise Mach number of 0.75. Next, as a baseline geometry, a transonic wing was taken from a previous research [9]. The original wing has an aspect ratio of 9.42, a taper ratio of 0.246 and a sweep angle at the quarter chord line of 23.7 deg. Its airfoil sections are supercritical and their thickness and twist angle distributions are reduced toward the tip. Then, only two parameters are chosen as design variables: sweep angle and taper ratio.

The objective functions and constraints are computed as follows. First, drag is evaluated, using a potential flow solver called FLO27 [10]. The code can solve subsonic and transonic flows. From the flow field solution, lift and drag can be post-processed. Since the flow is assumed inviscid, only a sum of the induced and wave drag is obtained. Second, wing weight is calculated, using an algebraic weight equation as described in Torenbeek [11]. Third, the fuel weight is calculated directly from the tank volume given by the wing geometry. Finally, the structural model is taken from Wakayama et al. [12]. In this research, the wing box is modeled only for calculating skin thickness. Then the wing is treated as a thin-walled, single cell monocoque beam to calculate stiffness. Flexibility of the wing is ignored. The objective function values and constraints' violations are now passed on to the system-level optimizer. MOGA is employed as the system-level optimizer here. When any constraint is violated, the rank of a particular design is lowered by adding 10.

In this section, the elitist model was frozen to BN and the results were compared between two niching models, FS and CSN. Figure 7 shows the resulting Pareto front obtained from BN + FS. BN + CSN gave a similar Pareto front and thus the result is not presented here. The major difference of the two, however, appears in the convergence history. As shown in Fig. 8, FS was able to converge the population to the Pareto front, but CSN was not. This is probably because of the adaptive σ_{share} used in FS. This result again suggests a need of an adaptive d_{min}. Figure 9 shows wing planform shapes of the resulting Pareto solutions. The extreme Pareto solutions are physically reasonable and the center of the Pareto front gives a good compromise.

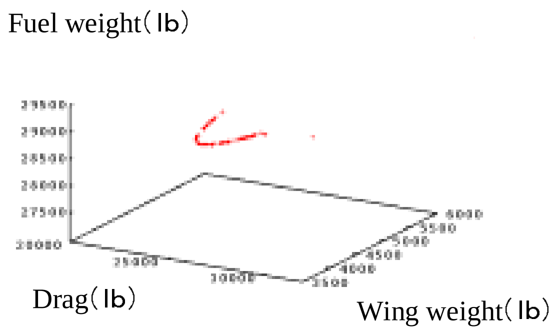

Fig. 7. Pareto solutions in objective function space.

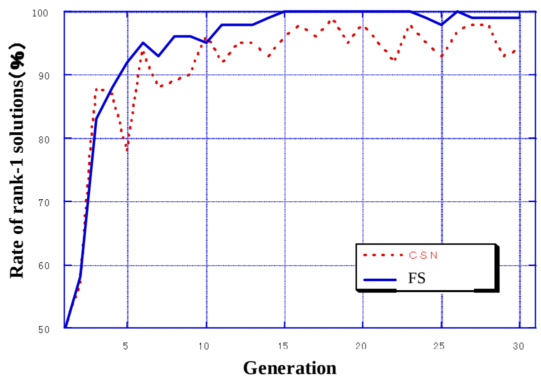

Fig. 8. Convergence history.

DRAG MINIMUM

CENTER OF PARETO

WING WEIGHT MINIMUM
FUEL VOLUME MAXIMUM

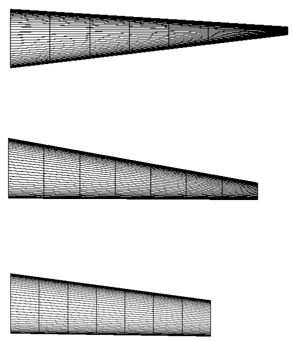

Fig. 9. Planform shapes of Pareto solutions.

Supersonic Wing Planform Design

To show the applicability of MOGA to supersonic wing planform design, the next MDO problem considers to

1. Minimize aerodynamic drag
2. Minimize wing weight
3. Minimize aspect ratio for structure

under a geometric constraint of the semispan-to-length ratio.

The definition of the supersonic wing planform geometry is also simplified here. The planform parameters were assumed as the semispan-to-length (lifting length of the wing) ratio of 0.45 and the root chord of 14.3 *ft* at cruise Mach number of 2.0. A symmetric airfoil section was assumed. Then, only four parameters are chosen as design variables: inboard and outboard sweep angles, chord length of the kink, and spanwise location of the kink. The tip chord length can be calculated from the specified parameters. These parameters can still produce a wide variety of planform shapes.

The objective functions and constraint are computed as follows. First, drag is evaluated, using the linearized theory for supersonic flows [13]. Second, wing weight is calculated, using the transonic algebraic weight equation [11]. The weight formula will be upgraded to a more adequate model for supersonic wings in future. Third, the aspect ratio is used instead of evaluating the structure, assuming that a lower aspect ratio provides stronger stiffness. Only the BN + FS was used.

Figure 10 shows the Pareto front in the objective function space and the planform shapes of the extreme Pareto solutions. The planform shape which gives the minimum drag has the largest aspect ratio. It also has a small wing area, and thus it is similar to the minimum wing-weight design. One of the compromised solutions is given by the center of the Pareto front. It tries to minimize the drag as well as to minimize the aspect ratio. Although the present disciplinary models are too simple to produce realistic designs, the extreme Pareto solutions are physically reasonable. These results confirm the feasibility of the present approach for solving MDO problems of aircraft wing planform shapes.

5. CONCLUSION

Niching and elitist models have been examined for multiobjective Genetic Algorithms (MOGAs). The fitness sharing and coevolutionary shared niching models were considered for niching. The simple generational model, elitist recombination, and the best-N selection were compared as the elitist model. The test results indicate that the combination of the fitness sharing and the best-*N* selection provides the best performance for MOGAs so far.

The resulting MOGA has been applied to MDO problems of transonic and supersonic wing planform shapes successfully. The extreme Pareto solutions are found physically reasonable and the center of the Pareto front gives a good compromise. The results confirm the feasibility of the evolutionary approach for aircraft design optimization.

6. ACKNOWLEDGMENT

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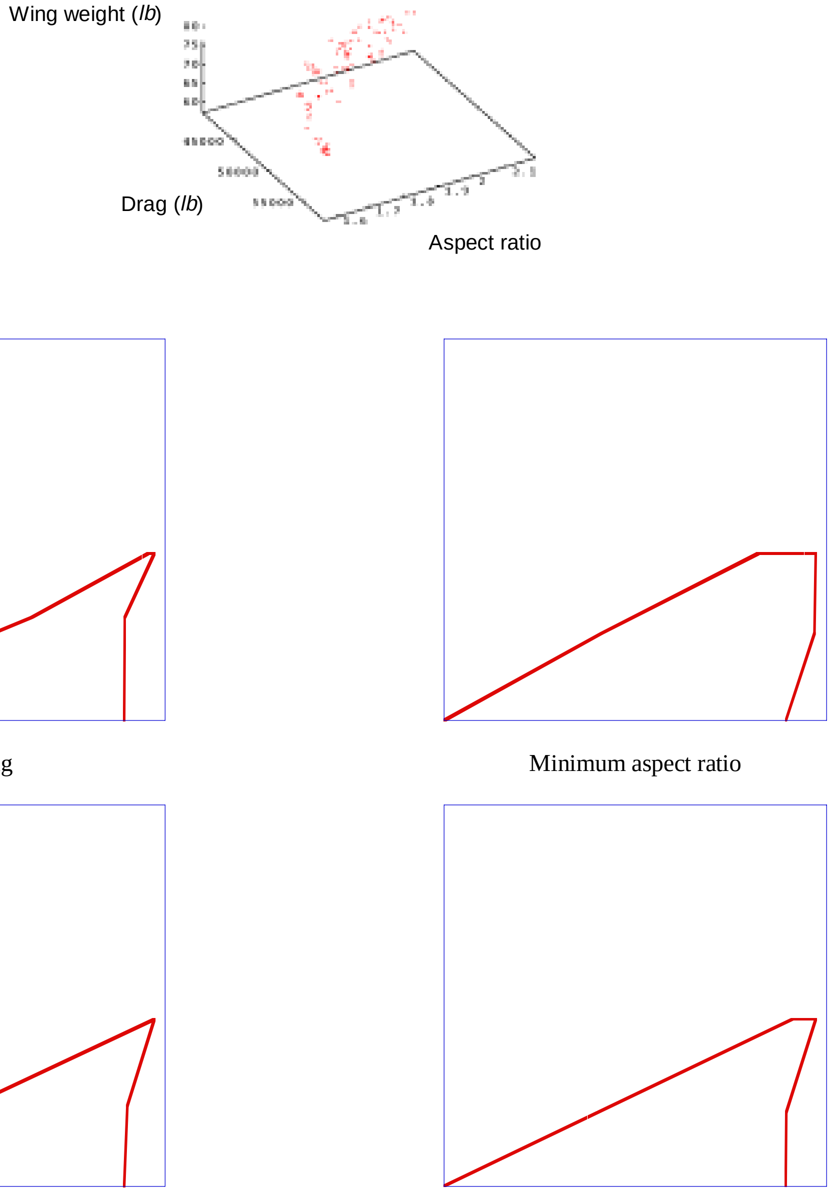

Fig. 10 Pareto front and extreme Pareto solutions.