

# Measurement of density distribution in a small cell by digital phase-shift holographic interferometry

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## ABSTRACT

Using digital phase-shift holographic interferometry (DPSHI), the internal density distribution of water which is sealed in a cylindrical cell has been measured under the condition that one of the cylinder end-wall is oscillated by a PZT-actuator, which is driven by high frequency:20kHz. The density distributions concerned with three different end-wall-pressure has been visualized and analyzed by DPSHI. Carré algorithm is employed for phase interpolation to reproduce phase maps. The absolute density changes in the cell has been demonstrated. The results have indicated that DPSHI can be allowed us to measure small density changes in narrow space under high frequency oscillation.

**Keywords:** digital image processing, phase shift holographic interferometry, sealed liquid, PZT-actuator

## 1. INTRODUCTION

The Shock Wave Research Center of the Tohoku University at Sendai, Japan, has been engaged for more than 20 years in the study of shock wave phenomena by primarily using holographic interferometry as a diagnostic tool. The studies range from fundamental investigations of the mechanisms of shock wave propagation and interaction to the implementation of shock wave technology to industrial and medical applications.

Double-exposure holographic interferometry has been extensively used in the experimental investigation of compressible flows, as this method possesses several advantages over other optical flow visualization methods. One of the most important aspects of this technique is that it can be utilized for the quantitative measurement of unsteady compressible flows, as those generated by shock waves.<sup>1</sup>

For a variety of applications, however, the sensitivity of such a conventional holographic interferometer is too low. For this reason, flow fields with very weak density variations cannot be adequately visualized and quantitatively evaluated. This holds true for all flows that are associated with shock Mach numbers  $M_S$  approaching unity.

Holographic interferometry is a special form of so-called reference beam interferometry,<sup>2</sup> where one fringe shift corresponds to a density increment  $\Delta\rho$  of

$$\Delta\rho = \frac{\lambda}{KL}, \quad (1)$$

where  $\lambda$ ,  $K$ , and  $L$  are the wavelength of light source, the Gladstone-Dale constant of the investigated gas, and the width of test section, respectively.<sup>1</sup>

Visualization and quantitative evaluation by conventional holographic interferometry may become difficult if the maximum density variation to be expected in the flow field does not exceed the density increment given by equation (1). Likewise, an evaluation of any density field with an accuracy of much less than the density increment corresponding to one fringe shift may become problematic. Special techniques have to be applied to increase the sensitivity of the interferometer, so that flow phenomena associated with density variations below the fringe shift increment can be made visible and quantitatively evaluated with sufficient accuracy. It should be stressed that the latter aspect is the main motive for the enhancement of these interferometric techniques, as pure visualization of weak density fluctuations may also be accomplished by other density-sensitive visualization techniques such as schlieren

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or shearing interferometry.<sup>3</sup> These methods, however, do generally not allow one to actually measure the density profiles.

Digital phase shift holographic interferometry is one of the methods that can be used for the desired sensitivity enhancement.<sup>4,5</sup> By using digital image acquisition techniques to record the reconstruction of double exposure holographic interferograms, one can determine phase distributions directly from a set of image irradiance measurements. This is possible, similar as in heterodyne holographic interferometry, if the phase of the interferogram can be manipulated in a known fashion. The results of these manipulations are then combined in order to evaluate the absolute phase value. The accuracy of this evaluation can be as high as 1/100 of a fringe shift.<sup>6,7</sup> In order to manipulate the phase of the interferogram, it has to be recorded with two reference beams.

## 2. EXPERIMENTAL

### 2.1. Recording system

Figure 1 shows the setup of the phase shift holographic interferometer. The system is essentially identical to a conventional holographic interferometer for diffuse illumination holograms. A double pulse ruby laser (wavelength  $\lambda = 694.3$  nm, 30 ns pulse duration, 2 J energy per pulse) is used as light source. The light beam is split into object beam and reference beam by means of a beam splitter. The object beam is expanded by a negative lens (focal length: -50 mm) before being scattered by a diffuser plate immediately in front of the test section. The reference beam is adjusted to have approximately the same optical path length as the object beam (accuracy: +/- 10 mm for a total path length of about 2.25 m). It is finally expanded by means of a negative lens (focal length: -25 mm) to completely fill the holographic recording plate.

The procedure of taking interferograms is the same as for conventional double pulse finite fringe interferograms. One holographic recording is taken of the test section before the flow starts, i.e., at known reference conditions, while the second exposure is initiated with the phase object, i.e., the flow, present in the test section. Between exposures, one mirror of the reference beam is slightly tilted by an angle of about  $0.05^\circ$ . In the present setup, this tilting was performed manually, which required a pulse separation of about 1 minute between exposures. By tilting the mirror, a second reference beam is generated, which is essential for the phase shift technique outlined above. The tilting of the mirror also produces a set of straight finite fringes,<sup>1</sup> whose location was generally found to be behind the fringes generated by the phase object.

### 2.2. Test section

Figure 2 shows the test section and the measurement system that are used in this experiment. Shown in Fig.3 is a picture of test section. Pure water which was sealed in a small cell: 7mm in diameter; 37mm in length, was oscillated by a piezo-actuator (PZT):  $17\mu\text{m}$  in travel distance; 800N in maximum generating force. The PZT was driven by 20kHz to generate pressure waves inside of the cell. A multi-function synthesizer (NF 1940) generated sine wave: 1V peak-to-peak; 20kHz, which was amplified up to 10 times with a high-speed power amplifier (NF 4025), then the amplified signal drove PZT. The end-wall pressure was measured with a pressure transducer (Toyota AA4200) which surface was level with the end-wall surface. A photo-diode was employed for detecting ruby laser irradiation to make certain that when the ruby laser irradiated. Trigger signal for starting measurement was generated with a function generator (NF FG-121). All of signals were recorded and stored in a digital sampling scope (Yokogawa DL-712).

### 2.3. Reconstruction system

Figure 4 shows the reconstruction setup that is used in this experiment. The system is essentially a Michelson interferometer, where two reconstruction beams with a small angular separation form a linear interference fringe pattern on the developed hologram plate. The angular separation of the reconstruction beams is adjustable so that the linear fringe pattern can be generated in such a way as to coincide exactly with the previously recorded pattern of finite fringes on the hologram. The superposition of these two linear fringe systems will produce a Moiré pattern except when the two fringe system exactly overlap. The purpose of introducing this Moiré technique is to eliminate misalignment errors caused by the fact that recording and reconstruction setup are physically different. Furthermore, this alignment allows one to compensate for the chromatic error that results from the wavelength difference between the recording and reconstruction lasers.<sup>8</sup>

In order to introduce phase shifting, one plane mirror of the Michelson interferometer is mounted on a piezo-translator, which can be displaced by distances equivalent to fractions of the laser wavelength. An objective lens

with low F-stop (F=1.4) was chosen for the recording CCD camera, and set close to the hologram so that the linear fringe pattern on the hologram was outside the image plane of the camera. Because of the aforementioned difference in the locations of the set of finite fringes generated by the tilting of the mirror and the fringes caused by the phase object, the finite fringes can be made to disappear from the image if the recording is performed with a low depth of field.

The camera output is digitalized in real time by a frame grabber, an 8-bit A/D converter with programmable look-up tables (LUT), and stored in a personal computer. Thus, during a videocycle, a digitized image is produced consisting of  $640 \times 480$  (VGA) pixels, where every pixel is quantified in 8 bits, that is, in 256 possible grey levels. This digitized image is then further processed according to image analysis and phase evaluation software.

In this study, Carré algorithm was employed for phase interpolation ( $0^\circ, 120^\circ, 240^\circ$ , and  $360^\circ$ ). For every step, the PC processed four irradiance values ( $I_1, I_2, I_3$ , and  $I_4$ ) at each point of the image, corresponding to a phase of  $\Phi, \Phi + 120^\circ, \Phi + 240^\circ$ , and  $\Phi + 360^\circ$ , respectively. Thus, the unknown phase distribution can be calculated from<sup>6</sup>

$$\Phi = \arctan \frac{\sqrt{[(I_1 - I_4) + (I_2 + I_3)][3(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}. \quad (2)$$

### 3. RESULTS AND DISCUSSION

Shown in Fig.3, are experimental results: (a) the reconstructed phase map; (b) the end-wall pressure profile and laser irradiation time; (c) the absolute density changes. In Fig.3, the condition at which the end-wall pressure was minimum value was analyzed. The phase distribution along the dotted line A-B in Fig.3(a) was converted into absolute density changes in such a way that combining Clausius-Mosotti equation<sup>2</sup> and the relation between fringe number and density changes gives us the equation:

$$N = \frac{L}{\lambda} \left( \sqrt{\frac{2K \cdot \rho + 1}{1 - K \cdot \rho}} - \sqrt{\frac{2K \cdot \rho_0 + 1}{1 - K \cdot \rho_0}} \right), \quad (3)$$

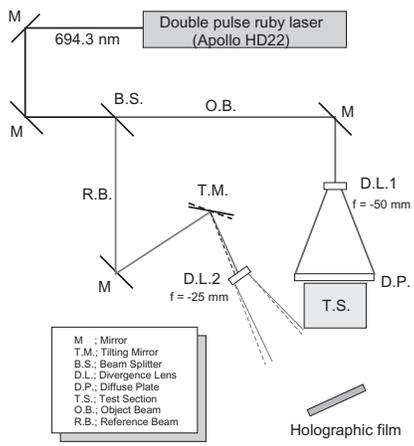
where  $N$  is fringe number,  $L$  is optical path length of test section,  $\lambda$  is laser wavelength,  $K$  is Gradstone-Dale constant,  $\rho$  is density and  $\rho_0$  is initial density.

### 4. CONCLUSION

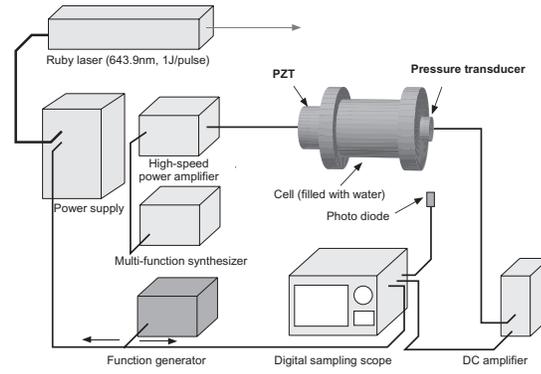
In the present work DPSHI has been applied for measurement of small density changes in a small cell which is filled with liquid and evaluated as a diagnostics technique for high speed phenomena in small space. This paper shows only experimental results but not numerical results therefore precision of DPSHI for this kind of phenomena was not discussed. This warrants future work on the numerical analysis of this experiment. Nonetheless, this experimental results have indicated that DPSHI for shock wave research can be applied for nanotechnology research.

### REFERENCES

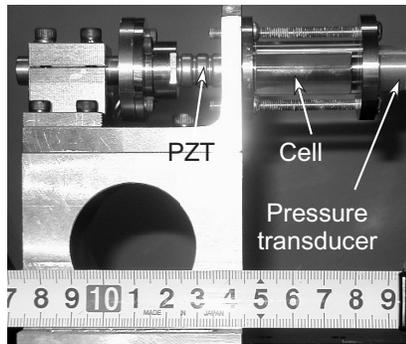
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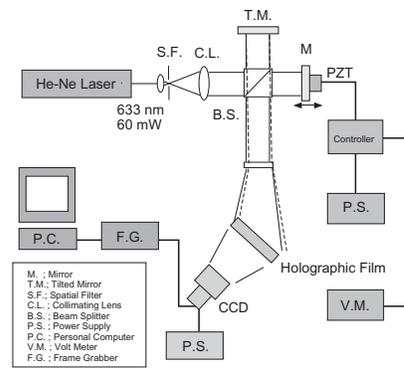
**Figure 1.** A schematic diagram of recording setup



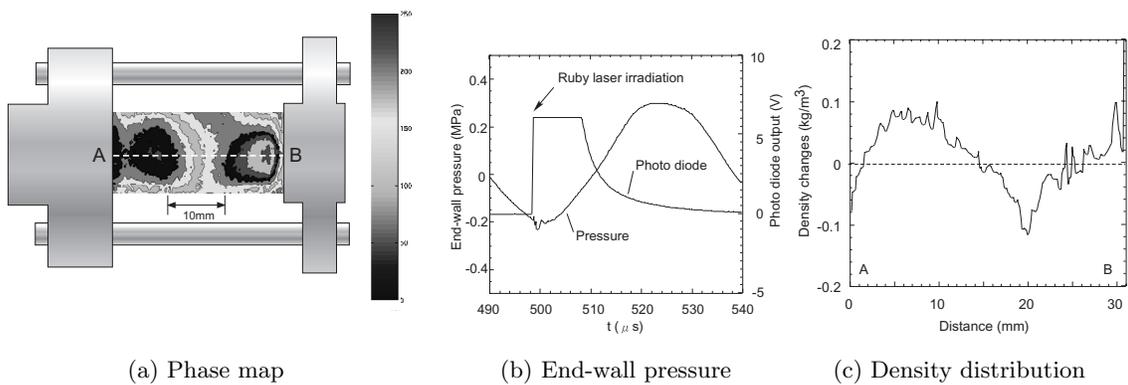
**Figure 2.** A schematic diagram of test section



**Figure 3.** A picture of test section



**Figure 4.** A schematic diagram of reconstruction setup



**Figure 5.** Results at minimum end-wall pressure