

Reality Coupled Computation for Multiphase Fluid Informatics

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We are mainly focus on the phenomenologically verified computation for the development of sustainable multiphase fluid machinery system which contributes to the green innovation. Research focus of our laboratory is to develop the frontier multiphase flow system, reality coupled computational method and integrated simulation based on high-speed laser optical measurement data.

7.1. Introduction

Our research group succeeded in the creation of a transdisciplinary integrated approach to energy science by incorporating the high functionality seen in phase transitions, chemical reactions, and electromagnetic forces into a multiphase flow in which the phases of several substances are mixed, as well as by developing new equipment that applies multiphase flow and an original analysis technique for multiphase flow dynamics. The achievements of our research group are due to a novel transdisciplinary integration approach based on fluidics, and greatly contribute to the construction of scientific principles for mechanical engineering and next-generation energy science.

7.1.A. Background

Cryogenic micro- and nano-solids are submicrometer-scale solid particles in an extremely low-temperature fluid. Our research group is focusing in particular on the high-functionality multiphase flow dynamics represented by the high enthalpy, ultra-high heat flux, and high density of high-velocity spray flow containing solid nitrogen micro- and nano-particles¹. The development of various types of next-generation equipment to be used in industry, such as new high-temperature superconductor cable cooling, next-generation processor cooling, high-density liquid rocket fuel, and high-enthalpy storage media will be made possible through solid-state nitrogen fine particles being stably supplied (Fig. 7.1). However, in the past only a solid flake production method the freeze-thaw/Auger method using vacuum drawing in a container was available as the conventional method for producing solid-state nitrogen. That existing method had the disadvantages of large particle size and the impossibility of continuous particle production. In light of that situation, our research group developed a new type of integrated numerical analysis technique for the creation of cryogenic solid-state nitrogen fine particles, and succeeded in the world's first experiments laying the foundation of technology for continuously producing solid nitrogen fine particles by means of a super-adiabatic two-fluid nozzle, solidifying the international reputation of our research group.

7.1.B. Research plan and principle

Solid nitrogen micro- and nano-particles are produced by a super adiabatic two-fluid nozzle method, developed by our research group, based on the principle of colliding subcooled liquid nitrogen and low-temperature helium gas (Fig. 7.1). In addition to developing a new type of integrated numerical analysis technique for the production of cryogenic solid-state nitrogen fine particles by this method (Fig. 7.1), our research group is developing the world's first technology for continuously producing solid-state nitrogen fine particles by means of a two-fluid nozzle¹. Furthermore, with respect to the present situation where no effective cooling method exists for next-generation processors with a high heat-generating density, our research group has proposed a new cooling method through the application of a micro- and nano-solid spray, and carried out ground-breaking experiments and investigations combining measurements and computational fluid dynamics in regard to the ultra-high heat flux refrigerant performance of a micro-solid nitrogen spray.

7.2. Originality and impact of research achievements in multiphase science

The solid-liquid or solid-gas system using a micro-solid phase is applied in micro- and nano-solid cooling and semiconductor cleaning method that our research group has succeeded in developing are world-first, original technologies; this research is in a field for which market expansion greater than that of gas-liquid system micro bubbles is expected in the future. The ultra-high heat flux electronics cooling system using micro-solid spray which our research group has developed is a groundbreaking idea that elegantly overcomes future key issues associated with next-generation processor cooling, and its effectiveness is exemplified below. First, the system possesses originality as

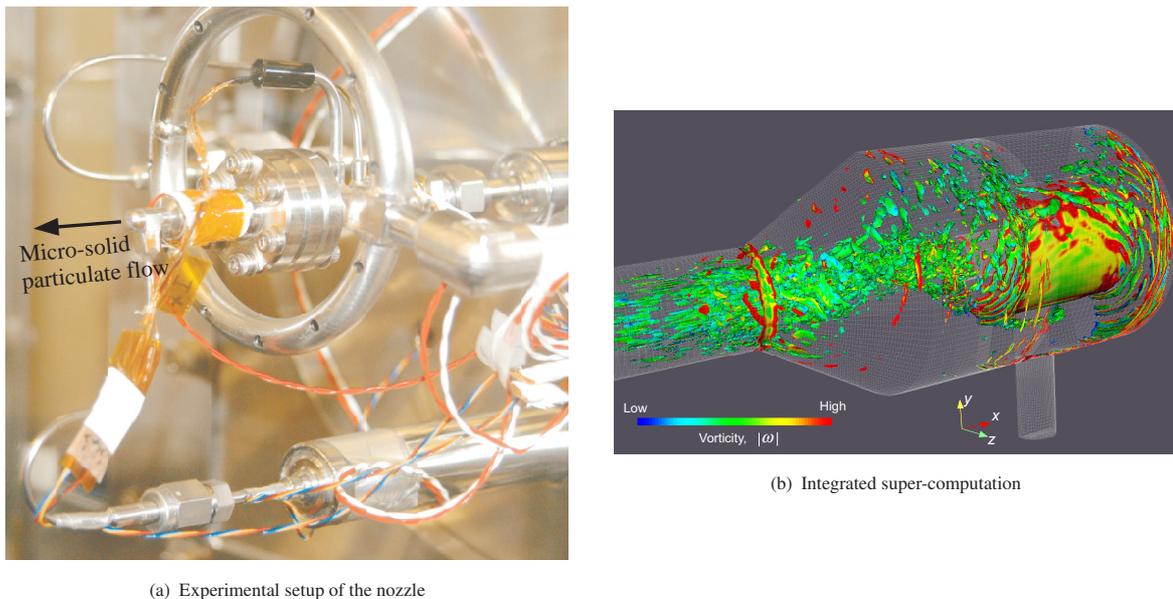


Figure 7.1. Superadiabatic two-fluid nozzle for cryogenic solid nitrogen particle production

follows: 1) Heat transfer characteristics are improved by using solid-state nitrogen fine particles at a temperature of 63 K-lower than that of liquid nitrogen (77 K) as the refrigerant. 2) Low-cost cooling with high-density, low-flow rates is possible because the density and the cooling enthalpy of solid-state nitrogen is 16% and 22% that of liquid nitrogen respectively. 3) Heat transfer and cooling characteristics can be realized that are several tens of times better than in the case a cooling method by heat transfer using forced water convection boiling, which currently has the highest cooling function.

Furthermore, with regards to the micro- and nano-solid sprays in the development of the semiconductor cleaning system, our research group shows originality in having developed a dry ashingless cleaning system consisting of simultaneous resist removal and cleaning process mechanisms that remove and clean resists from the wafer surface without an ashing process. This is achieved by continuously producing solid-state nitrogen fine particles by the high-speed collision of subcooled liquid nitrogen with low-temperature helium gas (refrigerant) and colliding them with the resists on the wafer surface as a micro- and nano-slush jet spray flow, and also by the interaction of the resist thermal contraction effect due to the inertia of the particles, the thermohydrodynamic effect of the spray, and the ultra-high heat flux rapid quenching. The system also has originality in facilitating the reduction of particle impact damage to fine patterns and in particular to the wiring pattern of next-generation three-dimensional gate structures which is accomplished by not using CO_2 as is used in existing blast cleaning and instead using solid particles of N_2 of small (low-carbon) molecular weight. Expansion to indium tin oxide (ITO) film detachment that will enable the recycling of ITO film for solar batteries and touch panels is also expected. Our research group has given eight invited lectures in Japan and overseas on research results relating to the above high-functionality multiphase fluids, and is having a significant global impact on mechanical engineering system researchers. In addition, as results on the cryogenic micro-solid nitrogen formation method and results on liquid hydrogen jet atomization rank 11th for January-March 2009 and 18th for October-December 2008 in Cryogenics², ScienceDirect Top 25 Hottest Articles, and as results on the atomization phenomenon that accompanies microcavitation rank 8th in the Journal of Engineering for Gas Turbines and Power Top 10 Most Downloaded Articles in October 2010³, the research of our research group is clearly attracting attention internationally.

7.3. Outlook for the reactive multiphase science

Our research group is planning a cyclical energy system using cryogenic solid particles including hydrogen, and aims to construct a recovery system utilizing hydrogen storage alloys of micro/nano slush hydrogen and to apply the technology in various industries for high sustainability as a cyclical high-enthalpy fuel for fuel cell power generation. In other words, our research group's aim is to realize a society that consumes the smallest possible quantity of fossil fuel through the development of a system that forms the basis of a low-emission hydrogen energy cycle that suppresses CO_2 production to the utmost, which can be considered a contribution to the promotion of green innovation.

Application is particularly expected in MEMS microchannels, for which frequency of usage is considered high, as a low viscosity dissipation, low pressure loss and high-efficiency cryogenic MEMS cooling system by utilizing

the extremely low viscosity of the working fluid. Human iPS cells are problematical in being extremely delicate in comparison with other stem cells, and are difficult to maintain in a high-quality state. However, if a glass freezing method using micro/nano solid cooling maintains high-quality and if long-term freezing technology can be established, a considerably extensive contribution to industry is expected, and the technology developed by this research could then make a great contribution spanning a wide range of interdisciplinary industries including medicine, medical engineering, and the life sciences. The construction of a recovery system using hydrogen storage alloys of slush hydrogen used in cleaning and cooling for the final stage of the energy system, and utilization as a highly sustainable cyclical high-enthalpy fuel for fuel cell power generation is anticipated. The research results acquired will conceivably contribute greatly to future high-temperature superconductivity cooling technology and the expansion of usage of all cryogenic multiphase fluids. Moreover, the results could contribute to the downsizing of fuel cell batteries and the reduction of propellant storage systems for liquid fuel rockets, and give rise to a synergistic effect in multidisciplinary mechanical engineering and energy science.

7.4. Introduction for innovative cryogenic micro-solid technology

In the present study, we developed a new type of cooling system using micro-solid nitrogen (SN_2) spray which makes it possible to obtain an ultra-high heat flux cooling performance. Furthermore, we investigated the cryogenic heat transfer and flow characteristics of micro- SN_2 spray impinging on a heated substrate by measurement coupled with integrated computational study. The micro- SN_2 spray was composed of fine solid nitrogen particles produced by the high-speed collision of subcooled liquid nitrogen and cryogenic gaseous helium (cryogen) using a super-adiabatic two-fluid nozzle¹. In our previous research¹, the 3-D structure of LN_2 atomization and the fine SN_2 particle production behavior of an micro-solid inside the small domain of a two-fluid LN_2 – GHe nozzle were numerically investigated and visualized by a new integrated simulation technique. However, cryogenic SN_2 spray behavior outside of the nozzle and functional thermal characteristics of SN_2 particles have not been sufficiently investigated. Therefore, we mainly focused on the thermal and ultra-high heat transfer characteristics of micro-solid nitrogen (SN_2) spray cooling. If this type of micro- SN_2 utilizing technology is widely applied to solid hydrogen (SH_2), development of the high enthalpy solid hydrogen energy circulation system may become possible⁴⁻⁶. Furthermore, the utilization of cryogen for spray cooling would be a novel cooling scheme for superconducting power cables⁷.

In the present study, it was found that a high-speed spray flow of a cryogenic micro-solid, which consisting of fine solid nitrogen particles applied as refrigerant, makes it more possible to attain extensively high multiphase heat transfer characteristics than in single-phase liquid nitrogen spray. This special feature of micro-solid spray is based on the ultra-high heat flux cooling performance of solid nitrogen impingement. To consider fundamental characteristics of ultra-high heat flux spray cooling heat transfer, several factors of the wall surface heat transfer process of micro- SN_2 particles were investigated. These can be characterized as follows: 1) direct wall contact, 2) forced convection with high-speed collision, 3) latent heat transport by solid melting to vaporization.

7.5. Experimental study

7.5.A. Experimental apparatus and measurement conditions

The entire experimental system is shown in Fig. 7.2. In this experimental system, 1) unsteady ultra-high heat transfer characteristics and cooling performance of micro- SN_2 spray and 2) micro- SN_2 particle diameter profile and particle motion can be simultaneously measured.

A schematic and photo of the new superadiabatic two-fluid ejector spiral nozzle for generating micro- SN_2 particles is shown in Fig. 7.3. In the present study, the principle of this nozzle developed in a previous study¹ was employed and a newly fabricated spiral device was placed at the aperture of the nozzle for enhancement of SN_2 atomization. The arrangement of the two-fluid ejector nozzle is as follows. When low-temperature gaseous He (GHe) stored in a tank as liquid helium (LHe) is injected into the ejector at high pressure and high-speed, subcooled pressurized liquid nitrogen (LN_2) is pumped up into the ejector from a LN_2 chamber, where it impinges on and mixes with the high-speed GHe flow. As a result, ultra-fine solid nitrogen (SN_2) particles are formed and ejected from the ejector nozzle aperture.

7.5.B. Measurement of ultra-high heat flux of micro- SN_2 spray

To clarify the effect of heat transfer characteristics of SN_2 spray on a heated substrate (19), unsteady ultra-high heat flux data of micro- SN_2 spray impinging on a heated substrate surface were measured. The data were compared with those of liquid nitrogen (LN_2) spray. After the substrate surface was heated until the determined temperature by a ceramic heater (20), SN_2 spray or LN_2 spray was emitted onto the heated substrate surface. The distance between nozzle aperture (5) and substrate surface in Figs. 7.3 was 2.5 mm. The transient heat flux was measured by cryogenic

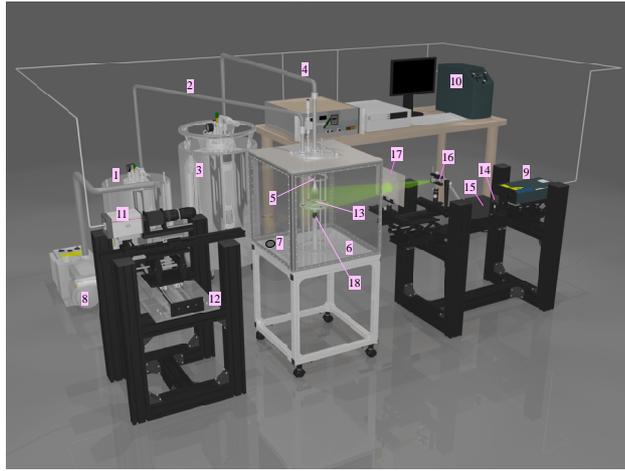


Figure 7.2. Schematic of the entire experimental system

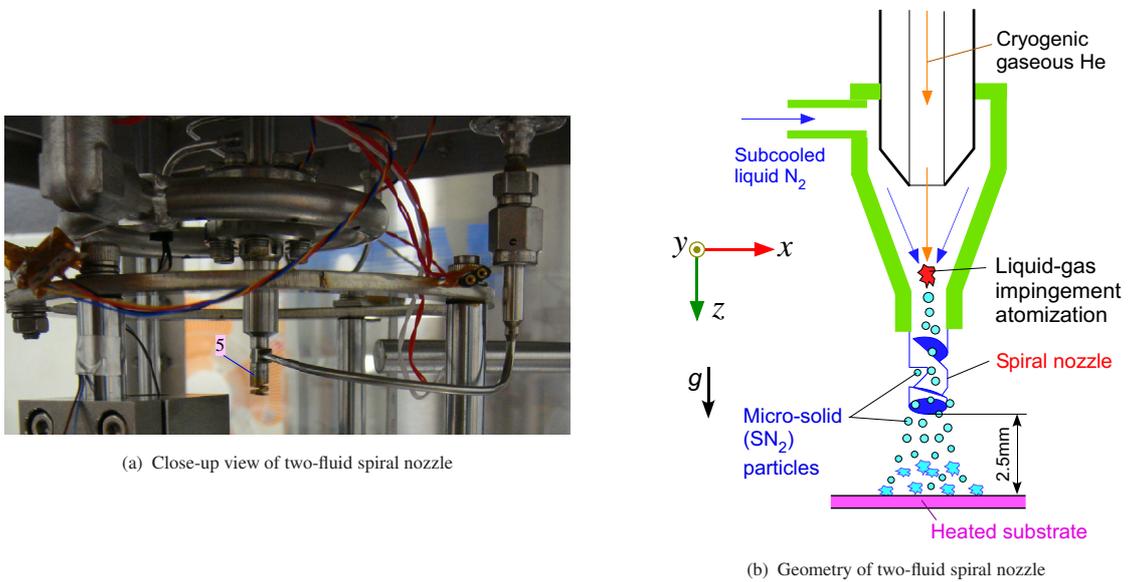


Figure 7.3. Aspects of the two-fluid ejector spiral nozzle for generating micro-SN₂ particle

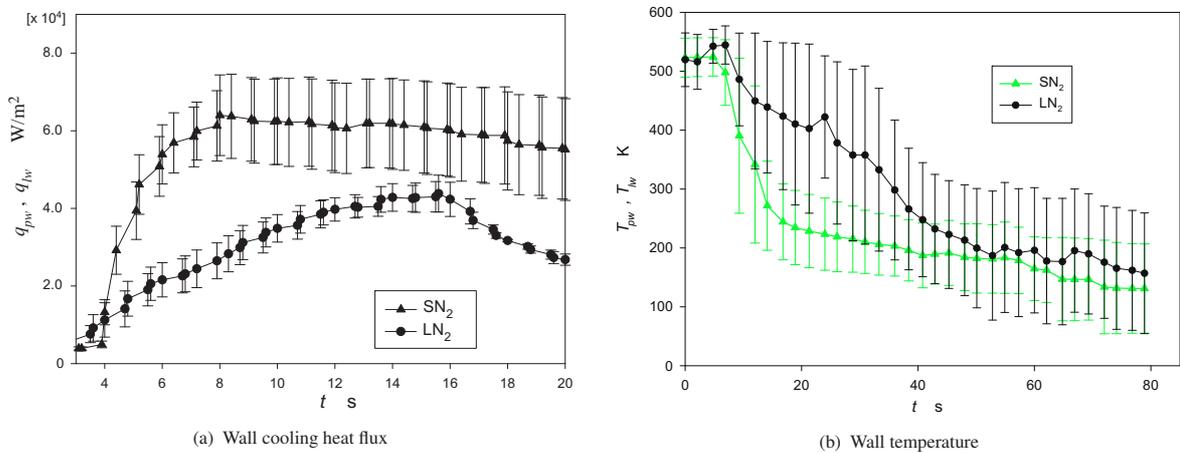


Figure 7.4. Transient measurement data of wall cooling heat flux and temperature decrease by emission of SN₂ and LN₂ spray

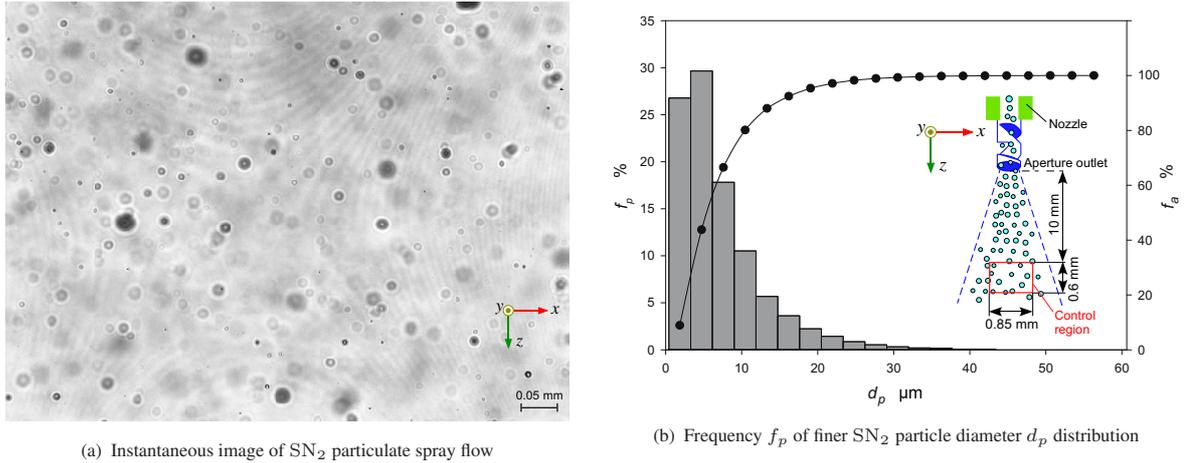


Figure 7.5. Experimental results for microscope probe assembled PIA-PTV

heat flux sensor (21).

Figure 7.4 shows the transient measurement data of the wall cooling heat flux the transient measurement data of the wall temperature by emission of SN_2 spray q_{pw} and LN_2 spray q_{lw} on the heated substrate. Also shows the transient measurement data of the wall temperature by emission of each spray case, T_{pw} and T_{lw} on the heated substrate. As for the error bars in the figures, they were generated on the basis of the standard deviation of the measurement data obtained several times, which were done under identical experimental conditions. The maximum cooling heat flux level of $1.1 \times 10^5 \text{ W/m}^2$ was obtained by present system. In the case of micro- SN_2 spray, the heated substrate of 550 K was rapidly cooled down to a T_{pw} of 200 K within only 8 seconds which is the most rapid temperature decreasing rate during measurement value. This temperature decreasing rate of SN_2 spray corresponds to about half the cooling time of LN_2 spray to achieve the same cooling temperature level. When micro SN_2 spraying is used, an ultra-high cooling heat flux approximately at a level of 10^5 W/m^2 is achieved during operation, a cooling performance 1.5 times that of LN_2 spraying. As SN_2 cooling has the advantage of direct latent heat transport which avoids the film boiling state, the ultra short time scale heat transfer in quite a thin boundary layer is possible in SN_2 but not LN_2 spraying. To obtain ultra-high heat flux cooling, the most important advantages of the SN_2 spray are the utilization of the direct contact heat transfer of SN_2 particles, and the utilization of latent heat transport with phase change, which avoids the film boiling state.

7.5.C. PIA-PTV optical measurement of SN_2 particulate flow

To clarify the effect of micro SN_2 particle behavior on ultra-high heat flux cooling performance, PIA (Particle Image Analyzer)-Shadow measurement was conducted⁸. The PIA-Shadow system is a special optical system which combines double pulsed laser illumination (Nd-YAG) and an enlarged visualization system. In the present measurements, by using a hybrid PIA-PTV (Particle Tracking Velocimetry) technique, the SN_2 particle diameter and motion (velocity) are quantized with a limited local region observation technique without disturbance of the moving particles group. LN_2 particles dispersed in the spray flow are illuminated by double pulsed flashes generated by an Nd-YAG laser. Particle images silhouetted by back lighting are observed simultaneously by a high-speed CCD camera. Focusing on SN_2 particle diameter and motion, PIA-PTV measurement is conducted without the thermal effect of a heated substrate.

Figure 7.5 shows the frequency f_p of finer SN_2 particle diameter d_p distribution in the dense particle number density N_p region obtained by microscope probe PIA analysis. The dimensions of the control region for PIA measurement are $0.6 \text{ mm} \times 0.85 \text{ mm}$, a smaller domain than the previous dilute case. In the vicinity of the dense spray core region just downstream of the nozzle aperture, it is found that the finer SN_2 particles, whose diameter is about $6.0 \mu\text{m}$, exhibit high frequency. It is also found that the mean diameter is about $7.08 \mu\text{m}$. From those PIA measurement results, SN_2 particle can be atomized to several micron meter orders by using the present micro- SN_2 spray production system. Especially in the dense spray core region, SN_2 particle atomization is enhanced by the shear stress acting on particles due to the high-speed axial spray velocity component. Therefore, finer particles are produced and exist with high probability in that region.

7.6. Computational study

7.6.A. Governing equations

To elucidate the ultra-high heat flux cooling heat transfer characteristics of micro-solid nitrogen (SN₂) spray using a two-fluid nozzle, integrated CFD analysis was carried out to clarify the cryogenic spray heat transfer mechanism that is difficult to obtain by conventional measurement. The numerical model represents the simultaneous unsteady cryogenic micro-SN₂ spray flow and heat transfer characteristics. Therefore, a new cryogenic spray model for analysis based on the unsteady thermal nonequilibrium two-fluid Eulerian-Lagrangian coupling model⁹ together with measurement data integrated CFD was applied to predict the micro-SN₂ spray flow characteristics. In this model, each phase is independently formulated by the basic equations. Eulerian equations are used to describe the behavior of the gas phase, comprising mixed gaseous helium (GHe) and gaseous nitrogen (GN₂). Also Lagrangian equations are used to describe the behavior of the dispersed SN₂ particle phase. The advantage of application of the Lagrangian model to the dispersed SN₂-phase is as follows: 1) each particle motion can be macroscopically handled as parcel motion, 2) momentum exchange between the particle phase and the surrounding gas-phases is precisely taken into account, and 3) several additional forces which act on particles can be considered by appropriate constitutive equations. The surrounding gas phase is assumed to be an averaged gaseous He (GHe) and gaseous N₂ (GN₂) state. Since the micro-SN₂ spray flow is considered to be a cryogenic turbulent flow, $k - \epsilon$ eddy viscosity model is used for the turbulent model.

To consider the effects of additional forces that act on the SN₂ particles, the equation of motion for the SN₂ particle phase is here replaced with the translational motion of a single particle, and given in the Lagrangian form¹⁰.

$$\frac{d}{dt} (\rho_p V_p \mathbf{v}_p) = -V_p \nabla p + V_p (\rho_p - \rho_g) \mathbf{g} + V_p \mathbf{F}_p, \quad (7.1)$$

where V_p is the volume of SN₂ particle, \mathbf{F}_p is the drag force which acts on the SN₂ particle. The thermodynamic properties of the particle are expressed in terms of JANAF temperature polynomials¹¹.

To estimate the order of each additional force term which appears in particle equation of motion (BBO equation, after Basset, Boussinesq and Oseen, who originally derived)⁹, the normalized form of BBO equation are introduced and several dimensionless parameter are derived.

7.7. Results and discussion

Figure 7.6 shows the instantaneous 3-D overall temperature (T) profile of spray and the characteristics of decreasing temperature of the heated substrate region. When the micro-SN₂ particles impinge on the substrate, the substrate is suddenly cooled down. It was numerically found that the ultimate high-heat flux cooling was attained by micro-SN₂ spray cooling. The satisfactory effect of decreasing temperature was obtained in the substrate surface region where the residual number density of impinging SN₂ particles on the substrate became large. This effective cooling performance is mainly caused by the wall contact heat transfer of SN₂ particle impingement and cryogenic forced convection due to the SN₂ inertia force. The most important advantageous factor of the SN₂ spray cooling system is the utilization of the direct contact heat transfer SN₂ particles and latent heat transport with phase change which avoids the film boiling state. Additionally, the effect of initial numerical conditions such as particle diameter, particle number per unit time, particle-phase velocity on the unsteady cooling heat flux were investigated.

Figure 7.8 shows the turbulent energy k profile of the overall cryogenic spray flow field and the micro-SN₂ particulate distribution for different initial particle diameters $d_{p(\text{in})}$ in the $x - z$ cross-sectional plane. The color contour in the surrounding fluid phase (gas-phase) shows the scalar magnitude of turbulent energy k , and the color contour in the particle phase shows the scalar magnitude of the SN₂ particle velocity. The particle image size in the figure corresponds to the magnitude of numerically predicted SN₂ particle size. The turbulent energy k was found to especially increase just under the nozzle aperture of SN₂ particle injection portion and to increase in vicinity of the heated substrate surface. The local increase of turbulent energy k was mainly caused by the frequent SN₂ particle-particle collision and atomization, which was extensively promoted just downstream of the spray injection portion. Furthermore, SN₂ particle-wall collision was promoted in the vicinity of the heated substrate surface. The magnitude of local turbulent energy k increased with the increase in initial particle diameter $d_{p(\text{in})}$. Especially, the increase of k close to the heated substrate was caused by inertial forces due to the large diameter size. In addition, since the collision reflection ratio of SN₂ particles on the heated substrate became large, an increase of k was induced. However, when the collision reflectance ratio of SN₂ particles was too large, contact wall heat transfer characteristics was deteriorated. Therefore, a method to control the SN₂ particle size distribution in order to manage the particle collision reflectance ratio which is based on the turbulent energy profile analysis is necessary.

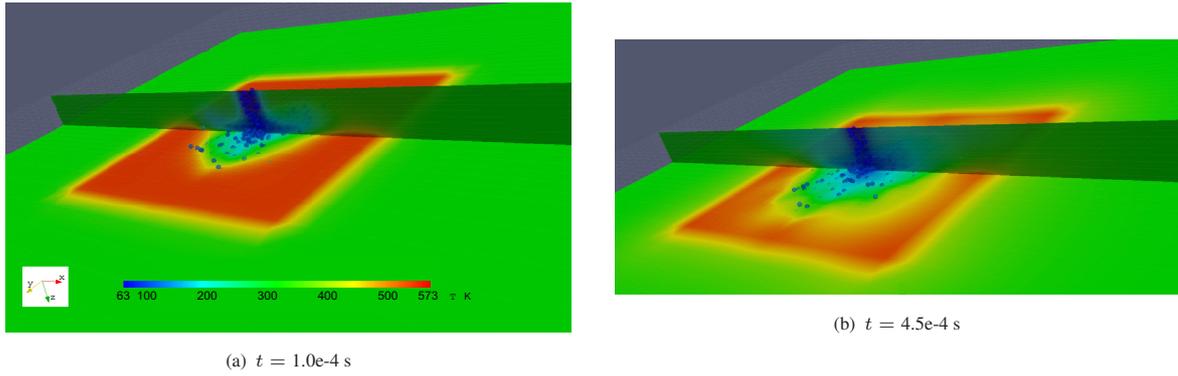


Figure 7.6. Instantaneous 3-D overall temperature T profile of spray and temperature decreasing characteristics of heated substrate region (Numerical: $d_{p(in)} = 5.0 \mu\text{m}$, $v_{p(in)} = 30.0 \text{ m/s}$, $N_{p(in)} = 3.0 \times 10^5 \text{ 1/m}^3$)

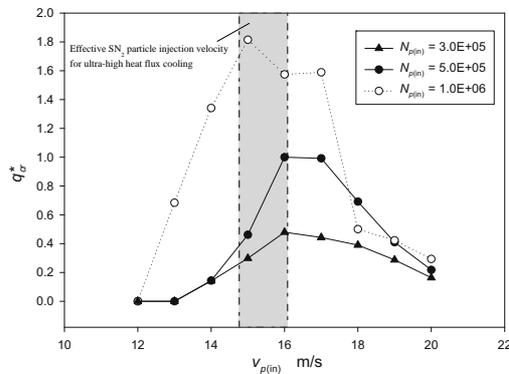


Figure 7.7. Effect of SN_2 particle injection velocity $v_{p(in)}$ on normalized critical cooling heat flux q_{cr}^* on the heated substrate surface under different initial particle number per unit time $N_{p(in)}$ conditions (Numerical: $d_{p(in)} = 5.0 \mu\text{m}$)

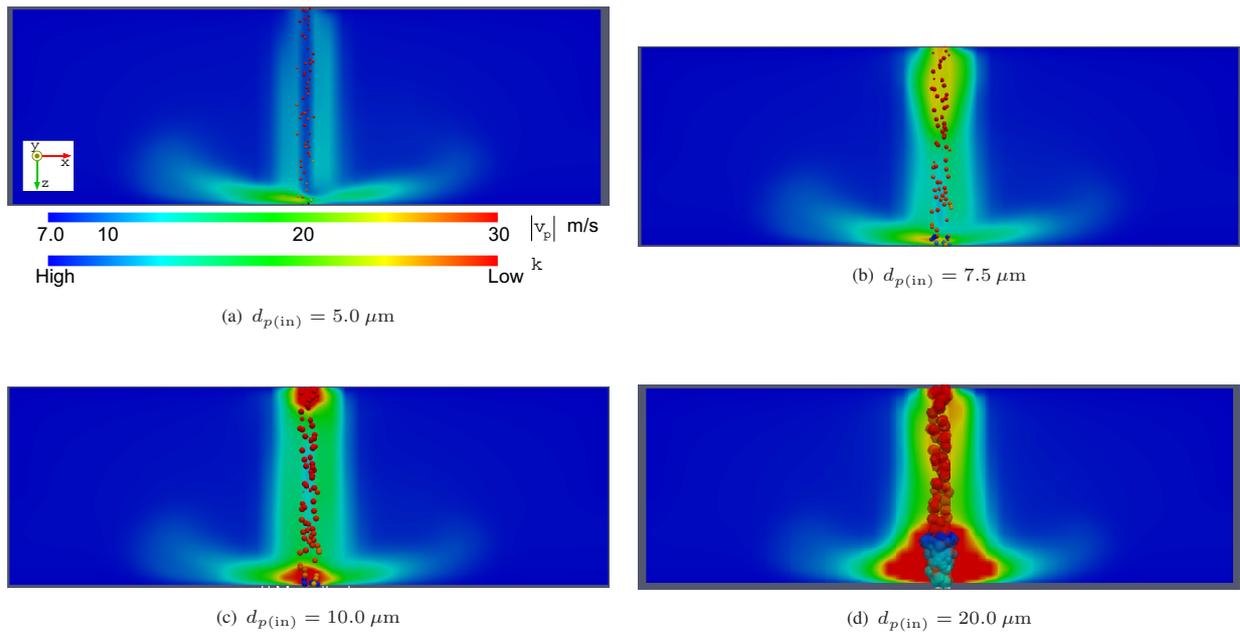


Figure 7.8. Turbulent energy k profile of overall cryogenic spray flow field, and micro- SN_2 particulate distribution under different initial particle diameters $d_{p(in)}$ in the $x-z$ cross-sectional plane (Numerical: $v_{p(in)} = 30.0 \text{ m/s}$, $N_{p(in)} = 3.0 \times 10^5 \text{ 1/m}^3$, in steady state)

7.8. Conclusion

The thermal and ultra-high heat transfer characteristics of micro-solid nitrogen (SN₂) spray cooling performance were precisely investigated by an integrated computational-experimental technique for the development of next generation super computer processor thermal management. The main results obtained can be summarized as follows.

1. When micro-SN₂ spraying cooling was used, an ultra-high cooling heat flux level was achieved in the effective cooling region during operation, much better than cooling performance of liquid nitrogen (LN₂) spray cooling. As SN₂ cooling has the advantage of direct latent heat transport which avoids the film boiling state, the ultra short time scale heat transfer in the quite thin boundary layer is more possible than that in the case of LN₂ spray. The present numerical prediction of micro-SN₂ spray cooling heat flux profile can reasonably reproduce the measurement result of cooling wall heat flux profiles.
2. The 3-D overall temperature profile of cryogenic spray and temperature decreasing characteristics of the heated substrate region were obtained by integrated computation. When the micro-SN₂ particles impinge on the substrate, the substrate is cooled suddenly. Ultimate high-heat flux cooling was numerically found to be attained by micro-SN₂ spray cooling. This effective cooling performance was mainly caused by the wall contact heat transfer of SN₂ particle impingement and cryogenic forced convection due to SN₂ inertia force. A satisfactory temperature decreasing effect was obtained in the substrate surface region where the residual number density of impinging SN₂ particle on the substrate becomes large.
3. The ultra-high heat flux cooling performance using micro-SN₂ spray was strongly affected by the initial particulate flow condition at the nozzle aperture. The initial SN₂ particle diameter, number density, and velocity influenced the wall contact heat transfer, forced convective heat transfer and latent heat transfer, which were closely related to the cryogenic atomization and spray flow characteristics. It was found that the numerically predicted optimized cooling condition reasonably agreed with the experimental results. These results show the validity of the present integrated numerical method. They also mean that the application of the numerically characterized initial setting value of cryogenic particulate spray to the configured experimental conditions effectively contributed to the enhancement of ultra-high heat flux cooling performance in actual operation.

Acknowledgments

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