Glassy Dynamics in Local Search by Metropolis Algorithm: Temperature-Cycling Experiments on Traveling Salesman Problems

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Glassy dynamics in local search by the Metropolis algorithm [1, 2] is investigated by temperature-cycling experiments on the traveling salesman problems (TSPs) [3]. This investigation is motivated by our recent findings on a positive role of glassy slow relaxation dynamics in optimization by the Metropolis algorithm and by an adaptive type of simulated annealing [4, 5]. A better understanding of this kind of search dynamics from the viewpoint of glasses is considered to be helpful in an effective design of local search optimization algorithm as well as a clarification of an appropriate analogy between complex physical systems and optimization problems.

We consider the random Euclidean TSP [3]: the location of the N cities are sampled uniformly in a unit square and the cost function is given by the tour length computed under the Euclidean metric. We use a 2-opt neighborhood [3] and the trial solution is randomly selected from the neighborhood.

The cycling process is controled in the following way. A randomly constructed tour is taken as an initial solution and the algorithm is employed with a fixed temperature T (the first stage). After the initial L search steps, which is measured by the number of cost evaluations, the temperature is changed to and maintained at $T+\Delta T$ (the second stage). Then, after the next L steps, the temperature is changed back to and fixed again at T (the third stage).

Besides actual search dynamics, we observe hidden search dynamics using a mapping-onto-minima method [6, 7]. The solution space is divided by the basin of the locally optimal solution and a transition process from basin to basin is observed as a temporal behavior of the cost of the inside locally optimal solution. We briefly describe this cost as the local minimum cost. We observe the histories of two costs, the actual cost and the local minimum cost, each of which is the average of 50 independent search processes. The glassy state of the present system is specified by the appearance of a logarithmic decay in the average standard relaxation curve of the local minimum cost [4]. The simple local search used in the mapping-onto-minima process is executed by means of the best admissible search strategy.

The experiments are performed for the size $N = 100$ under the various combinations of the values of L, T, and ΔT. Our findings are summarized as follows: (i) A Memory effect [8, 9], which has been observed in various glasses for negative cycles ($\Delta T < 0$), is observed also in the present random TSP instance at low temperatures: the effect is observed in both of two average histories. (ii) For positive cycles ($\Delta T > 0$), the average history of the actual cost deviates downward from that of the standard relaxation curve at the third stage. At this point, the present TSP instance has a resemblance to polymer glass [8], not to spin-glass [9]. (iii) For positive cycles, the relaxation of the local minimum cost is accelerated at the second stage. This acceleration seems to affect the optimization performance in the present experimental time scale ($\sim 10^7$ search steps).

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