First-order phase transition of tethered membrane models

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We find a first-order transition separating the smooth phase from the crumpled one in two kinds of tethered surface models [1, 2]. The canonical Monte Carlo (MC) simulations were carried out on triangulated spherical surfaces. Experimentally, a phase transition that is very similar to the transition in this paper has been detected recently [3].

The first model (model 1) is defined by the Hamiltonian which is a linear combination of the Gaussian term $S_1$ and the bending energy term $S_2$ such that

$$S = S_1 + b S_2, \quad S_1 = \sum_{(i,j)} (X_i - X_j)^2,$$

$$S_2 = \sum_{ij} (1 - n_i \cdot n_j),$$

where $\sum_{(i,j)}$ denotes the sum over bond $(i,j)$ linking the vertices $X_i$ and $X_j$, $\sum_{ij}$ is the sum over triangles $i,j$ sharing a common bond, and $b$ is the bending rigidity. The symbol $n_i$ in Eq. (1) denotes the unit normal vector of the triangle $i$.

The Hamiltonian of the second model (model 2) is given by a linear combination of the bending energy term $S_2$ in Eq. (1) and a hard-wall potential $V$ such that

$$S = S_2 + bV, \quad V = \sum_{(ij)} V(|X_i - X_j|),$$

$$V(|X_i - X_j|) = \begin{cases} 0 & (0 < |X_i - X_j| < r_0) \\ \infty & (\text{otherwise}) \end{cases},$$

where $r_0$ gives the upper bound on the bond length and is chosen as $r_0^2 = 1.1$.

The standard Metropolis MC technique is used to update the variables $X$. The surfaces are obtained by dividing the icosahedron, and hence, are uniform in the co-ordination number.

Figure 1(a) is the variation of $S_2/N_B$ against MCS (Monte Carlo sweeps) of the model 1 on the $N = 10242$ surface at the transition point, where $N_B$ is the total number of bonds. The distribution (or histogram) $h(S_2)$ of $S_2/N_B$ is plotted in Fig. 2(b). We find a discontinuous transition in the model 2 from the double peak structure in $h(S_2)$.

Figure 2(a) is the variation of $S_2/N_B$ against MCS of the model 2 on the $N = 8412$ surface at the transition point. The distribution (or histogram) $h(S_2)$ of $S_2/N_B$ is plotted in Fig. 2(b). We find a discontinuous transition in the model 2 from the double peak structure in $h(S_2)$.

References

