First-order phase transition of tethered membrane models

Hiroshi Koibuchi
Ibaraki College of Technology
Nakane 866, Hitachinaka, Ibaraki-312-8508, Japan

We find a first-order transition separating the smooth phase from the crumpled one in two kinds of tethered surface models [1, 2]. The canonical Monte Carlo (MC) simulations were carried out on triangulated spherical surfaces. Experimentally, a phase transition that is very similar to the transition in this paper has been detected recently [3].

The first model (model 1) is defined by the Hamiltonian which is a linear combination of the Gaussian term S_1 and the bending energy term S_2 such that

$$S = S_1 + bS_2, \quad S_1 = \sum_{(i,j)} (X_i - X_j)^2 ,$$
$$S_2 = \sum_{i,j} (1 - \mathbf{n}_i \cdot \mathbf{n}_j) , \qquad (1)$$

where $\sum_{(i,j)}$ denotes the sum over bond (i,j) linking the vertices X_i and X_j, $\sum_{i,j}$ is the sum over triangles i,j sharing a common bond, and b is the bending rigidity. The symbol $\mathbf{n}_i$ in Eq. (1) denotes the unit normal vector of the triangle i.

The Hamiltonian of the second model (model 2) is given by a linear combination of the bending energy term S_2 in Eq.(1) and a hard-wall potential V such that

$$S = S_2 + bV, \quad V = \sum_{(ij)} V(|X_i - X_j|), \qquad (2)$$
$$V(|X_i - X_j|) = \begin{cases} 0 & (0 < |X_i - X_j| < r_0) \\ \infty & (\text{otherwise}), \end{cases}$$

where r_0 gives the upper bound on the bond length and is chosen as $r_0^2 = 1.1$.

The standard Metropolis MC technique is used to update the variables X. The surfaces are obtained by dividing the icosahedron, and hence, are uniform in the co-ordination number.

Figure 1(a) is the variation of S_2/N_B against MCS (Monte Carlo sweeps) of the model 1 on the $N = 10242$ surface at the transition point, where N_B is the total number of bonds. The distribution (or histogram) $h(S_2)$ of S_2/N_B is plotted in Fig. 1(b). The double peak structure in $h(S_2)$ represents a discontinuity of S_2, and hence the existence of a discontinuous transition in the model 1.

Figure 2(a) is the variation of S_2/N_B against MCS of the model 2 on the $N = 8412$ surface at the transition point. The distribution (or histogram) $h(S_2)$ of S_2/N_B is plotted in Fig. 2(b). We find a discontinuous transition in the model 2 from the double peak structure in $h(S_2)$.

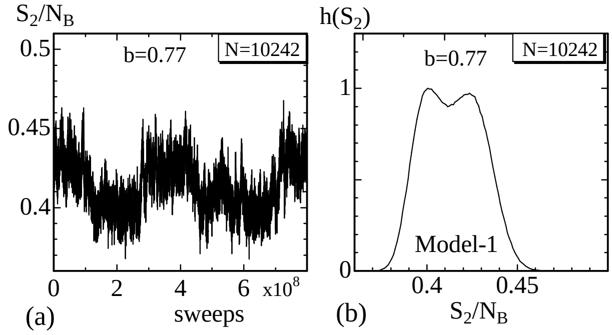

Figure 1: (a) Variation of S_2/N_B against MCS and (b) the histogram $h(S_2)$ of S_2/N_B, of the model 1, $N = 10242$, $b = 0.77$.

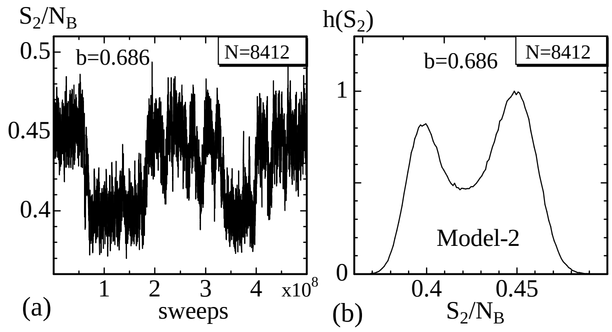

Figure 2: (a) Variation of S_2/N_B against MCS and (b) the histogram $h(S_2)$ of S_2/N_B, of the model 2, $N = 8412$, $b = 0.686$.

References

[1] H. Koibuchi, and T. Kuwahata, cond-mat/0506787, Phys. Rev. E72, *in press.*

[2] H. Koibuchi et.al., *submitted.*

[3] S. Chaieb, A talk in the workshop in Benasque Center for Sciences, "Biological Membranes: Current Challenges", (2005).