Generalized time-dependent harmonic oscillator at finite temperature

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We consider a generalized time-dependent harmonic oscillator (GTHO) whose Hamiltonian is given by

$$H(t) = A(t)p^2 + B(t)(xp + px) + C(t)p + D_2(t)x^2 + D_1(t)x + D_0(t),$$

where $A(t), B(t), C(t)$ and $D_i(t)$ $(i = 0, 1, 2)$ are time-dependent coefficients at finite temperature. The system described by the Hamiltonian becomes nonequilibrium (out of equilibrium) when coupling constants of the Hamiltonian change rapidly through the interaction with an environment. When we extend a certain mechanical system from zero-temperature to finite temperature we usually use the density operator assumed to be $e^{-\beta H}$, where H is a system Hamiltonian and β is an inverse temperature. However, the density operator is not given by $e^{-\beta H(t)}$ in GTHO since Hamiltonian $H(t)$ dosen't satisfy the Liouville von Neumann (LvN) equation. Instead we introduce a invariant operator $I(t)$ satisfying LvN equation

$$\frac{dI(t)}{dt} = \frac{\partial I(t)}{\partial t} + \frac{1}{i\hbar}[I(t), H(t)] = 0$$

as a conservative quantity in GTHO. Then the density operator is given by $e^{-\beta I(t)}$. The invariant operator is represented by creation and annihilation operators [1]. By using these operators, we can construct the time dependent Fock space in GTHO.

In this study we extend this system to finite temperature by using thermo field dynamics (TFD). The TFD introduced by Takahashi and Umezawa is a canonical formalism for finite temperature theory to describe quantum systems in thermal equilibrium [2]. In a simple terminology, the TFD of a quantum system doubles the degrees of freedom by introducing a fictitious Hamiltonian without any interaction with the system, and use an extended Hilbert space which is the direct product of the Hilbert space of the system and the fictitious system. In the oscillator representation there is a temperature-dependent Bogoliubov transformation between the annihilation and creation operators of the total system and those temperature dependent ones. Then the thermal state is a two-mode squeezed vacuum state of the extended Hilbert space, which can be annihilated by the temperature-dependent annihilation operators.

Here we show how to extend GTHO to finite temperature by using TFD. We derive the time-dependent annihilation and creation operators for the system described by $H(t)$ and the associated fictitious system, and find the time- and temperature-dependent annhilation and creation operators through temperature dependent Bogoliubov transformation of TFD. Then we also obtain the thermal state as a two-mode squeezed vacuum state in time-dependent case as in time-independent case.

References

[1] J. Choi, Pramana - J. Phys. **62** (2004) 13.

[2] Y. Takahashi and H. Umezawa, Collect. Phenom. **2** (1975) 55.