Delayed Random Walks and Control

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Delayed random walks have been proposed and studied as one approach to investigate systems with noise and delay[1]. This is a random walk whose transition probability depends on its position at a fixed interval in the past. The focus has been placed on a model that has an attractive bias to a single point. This stable case has been applied to such processes as posture controls[2]. Analytically, the attractive delayed random walk model has shown such behavior as an oscillatory correlation function with increasing delay.

However, such a model is not suitable to model an unstable situation, like balancing a stick on a fingertip[3]. Instead, a delayed random walk that has a repulsive point is used. We can consider many different possibilities, but here we test one-dimensional discrete time and step random walk with the origin as a repulsive point. Mathematically, we can define our model as follows. Let the position of the random walker at time step t be given by $X(t)$ and the fixed point be set at the origin, $X = 0$. The delayed random walk is defined by the following conditional probabilities.

$$\begin{aligned}
P(X(t+1) = X(t)+1|X(t-\tau) > 0) &= p \\
P(X(t+1) = X(t)+1|X(t-\tau) = 0) &= \frac{1}{2} \\
P(X(t+1) = X(t)+1|X(t-\tau) < 0) &= 1-p,
\end{aligned}$$

where $0 < p < 1$ and τ are the delay. The walker refers to its position in the past with delay to decide on the bias of the next step. The attractive delayed model is a case of $p < 0.5$, where the origin becomes attractive with no delay, $\tau = 0$. However, $p > 0.5$ gives the repulsive case that will be discussed in the rest of this paper. Though this appears to be a small change from the attractive delayed case, we actually observed very different behavior from it. Most of all, when the walker escapes from the origin, we no longer have a stationary probability distribution. This makes an analytical treatment of this repulsive model more difficult compared with the attractive delayed case, particularly with a non-zero delay. Our investigation in this paper was done using computer simulations. The most notable feature of this model is that we can find an optimal combination of the bias p and τ where the random walker can be kept around the origin for the longest duration. This may be another form of resonance with noise and delay[4].

These theoretical results imply that systems can reach a better balancing performance if an appropriate amount of fluctuation is added given the feedback or reaction delay. We have termed this type of control, which is different from standard feedback or predictive ones, as delayed stochastic control. We performed the following experiment to gain some insight into the existence or utilization of this control scheme. We asked the subjects to sit on a chair and balance a stick, as in the previous stick balancing experiment. But, this time, the subjects were allowed to move their bodies, not just their arms, as they tried to balance the stick. One way to do this is to hold an object with the other hand and move it. Another way is to move their legs. We measured the time for which they could keep the sticks balanced, and compared it with the normal non-movement situations. Out of the six subjects we tested, three subjects showed notable improvement in balancing by reaching their own optimal level of movement.

Some practice was needed for these subjects to reach this better performance. We believe that the subjects were tuning the appropriate level of fluctuation given their reaction times and prediction accuracy. Even though more thorough data needs to be collected, these results may be one supporting example of delayed stochastic control. We discuss this issue with video presentation of the above experiment.

References

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