Statistical-mechanical Approaches to the Problem of Atmospheric Compensation in Adaptive Optics

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In adaptive optics, wave-fronts generated by a reference source often carry information through the transmission channel. However, the information on the wave-fronts is not completely transmitted to a receiver due to noises, such as atmospheric disturbance and measurement errors. Therefore, in order to realize reliable wave-front compensation, it is important to construct technologies of wave-front reconstruction using wave-front slopes observed by optical instruments. Also, this problem has a difficulty that the conventional method is not successful, if "aliasing" occurs due to under-sampling. For this problem, the least-squares estimation [1] and its variants, the Bayesian inference based on the MAP estimate [2] have been attempted. On the other hand, based on the analogy between statistical mechanics of magnetic systems and information science, statistical mechanical methods [3] have been applied to problems, such as image restoration, error-correcting codes.

In this study, based on statistical mechanics of the Q-Ising model on the square lattice, we formulate the problem of wave-front compensation in adaptive optics. This method is based on the maximizer of the posterior marginal (MPM) estimate using the wave-front slopes corrupted by atmospheric disturbance and measurement errors. Then, our method is classified into two kinds according to the sensitivity of measurement as follows.

If the Nyquist condition holds at every sampling point, the wave-front is retrieved using the MPM estimate as

$$z_{x,y} = \arg\max_{zx,y} \sum_{\{x\}\neq} P(\{z\})P(\{J\} \mid \{z\}),$$

using the system $\{z_{x,y}\}(z_{x,y} = -R/2 + k_{x,y}R/Q, k_{x,y} = 1,\cdots,Q)$ at the lattice points (x, y), where we assume the models of noise probability and true prior as

$$P(\{A\} \mid \{z\}) \propto \exp\left[-\frac{J}{T_m}\sum_{n.n.}\left(A_{x,y}^{x',y'} - z_{x,y} + z_{x',y'}\right)^2\right],$$

$$P(\{z\}) = \frac{1}{Z_m}\exp\left[-\frac{1}{T_m}\sum_{n.n.}\left(z_{x,y} - z_{x',y'}\right)^2 + \frac{h}{T_m}\sum_{x,y} z_{x,y}\right],$$

where this model prior is considered to enhance smooth structures in the wave-fronts spreading in $z > 0$.

On the other hand, when "aliasing" occurs, we retrieve wave-fronts based on the MPM estimate using an initial wave-front constructed so as to minimize the difference of the wave-front slopes between neighboring sampling points.

Next, in order to clarify the performance of our method, we apply the Monte Carlo simulation to a typical wave-front in adaptive optics. Here, atmospheric disturbance is assumed to be Gaussian noise onto the original wave-front and measurement errors are assumed to be the complex Gaussian noise onto the wave-front slopes of the corrupted wave-front. As one of our results is shown in Fig. 1, when the Nyquist condition holds, we find that even the MPM estimate using the model prior with uniform distribution works well for phase retrieal. Then, as two examples of our results are shown in Figs. 2 and 3, we also clarify that the model prior expressed by the Boltzmann factor of the Q-Ising model under the uniform field improves the performance of the MPM estimate, if we appropriately set parameters. Next, if "aliasing" occurs, we derive the result that the MPM estimate works well if we start from an appropriate initial wave-front constructed so as to minimize differences of neighboring wave-front slopes.

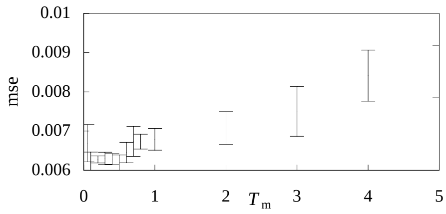

Figure 1: The mean square error as a function of T_m due to the MPM estimate for atmospheric compensation.

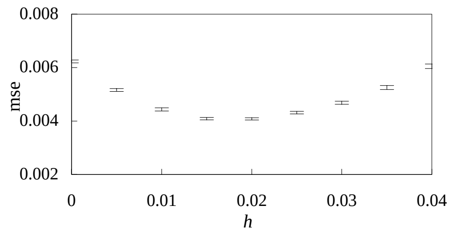

Figure 2: The mean square error as a function of h due to the MPM estimate for atmospheric compensation.

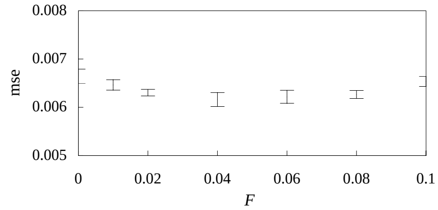

Figure 3: The mean square error as a function of F due to the MPM estimate for atmospheric compensation.

References

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