Dynamics of a Hogg-Huberman Model with Time-Dependent Reevaluation Rate

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There have been many efforts to investigate the dynamical behavior of multi-agent systems with distributed resource allocation in economic and social systems. Huberman and Hogg [1] studied the dynamics of resource allocation in a model comprised of many individual agents, including resource contention and decision making based upon incomplete knowledge and delayed information. They showed that in two resource system, periodic and chaotic oscillations appeared under certain conditions. Hogg and Huberman [2] have discussed a method of controlling chaos in the system composed of interacting agents making decision based on imperfect and delayed information. Hereafter we call this agent system the Hogg-Huberman model (H-H model). The system composed of multi-agents whose making decision for resource choice under the uncertain and delayed information is a kind of complex system. The dynamics of the H-H model with respect to discrete time has been discussed by several authors [3, 4]. In the H-H model, the rate at which agents reevaluate their resource choice does not depend on time and is regarded as a constant even though the state of the system has been changed. In the case of a discrete time, we have investigated the dynamical behavior of H-H model with a time-dependent reevaluation rate [5]. Now we are interested in the case of continuous time t. In the present work we study dynamics of the H-H model with time-dependent reevaluation rate.

Now we consider the Hogg-Huberman model with two resources. The dynamical equation for fraction $f_1(t)$ of agents using resource 1 at a time t may be expressed as

$$\frac{df_1(t)}{dt} = \left(\alpha_1(t) - \alpha_2(t)\right)\rho_1(t) f_1(t) + \alpha_2(t)\rho_1(t) - \alpha_1(t) f_1(t) \tag{1}$$

where $\rho_1(t)$ is the probability that an agent will prefer resource 1 to resource 2 when it makes a decision and is given by

$$\rho_1(t) = \frac{1}{2}\left[1 + erf\left(\frac{G_1(f_1(t-\tau)) - G_2(f_1(t-\tau))}{\sqrt{2}\sigma}\right)\right] \quad . \tag{2}$$

Here $erf(x)$ is the error function of x, σ quantifies the uncertainty of information, and τ denotes a time delay of conveying it. $G_1(f_1(t-\tau))$ and $G_2(f_1(t-\tau))$ are payoff functions for using resources 1 and 2, respectively. The reevaluation rate might depend on agent's behaviors in the system. When agents using one resource have information about the higher payoff for agents using another resource, most of agents getting lower payoff may change their resource choice. The reevaluation rates $\alpha_1(t)$ and $\alpha_2(t)$ for agents using resource 1 and 2, respectively, may be expressed as,

$$\alpha_1(t) = \lambda B_2(t) / \sum_{r=1}^{2} B_r(t) \ , \tag{3}$$

$$\alpha_2(t) = \lambda B_1(t) / \sum_{r=1}^{2} B_r(t) , \tag{4}$$

It is assumed here that the rate $\alpha_1(t)$ for agents using resource 1 is proportional to the actual payoff received by agents using resource 2 at time ($t-\tau$), and vice versa. λ is a factor of characterizing the resource evaluation. The actual payoff $B_r(t)$ received by agents using resource r at time ($t-\tau$) is defined by

$$B_r(t) = f_r(t-\tau)\, G_r(f_r(t-\tau)) \quad , (r=1, 2) \tag{5}$$

A time delay τ about information appears through payoff functions and affects agent's decision for resource choice. When $\alpha_1(t) = \alpha_2(t) = \alpha$, Eq. (1) is reduced to the dynamical equation of H-H model with a constant evaluation rate. Thus, we have obtained the dynamical equation of Hogg-Huberman model with the time-dependent reevaluation rates.

In order to investigate effects of the time dependent reevaluation rate on dynamics of the system, we have calculated time development of the fraction $f_1(t)$. It is found that the chaotic oscillation in the Hogg-Huberman model becomes the periodic oscillation in the case of time-dependent reevaluation rate. We have also shown the expression of $\alpha_1(t)$ and $\alpha_2(t)$ by means of the actual payoff received by agents using resource r at time ($t-\tau$). In numerical calculations, it is seen that the system performance may improved by introducing the time-dependent reevaluation rate, and controlling chaos in the system may be performed by the system itself.

References

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