A study on cluster-based reduced-order model for optimal feedback control of dynamic stall flow

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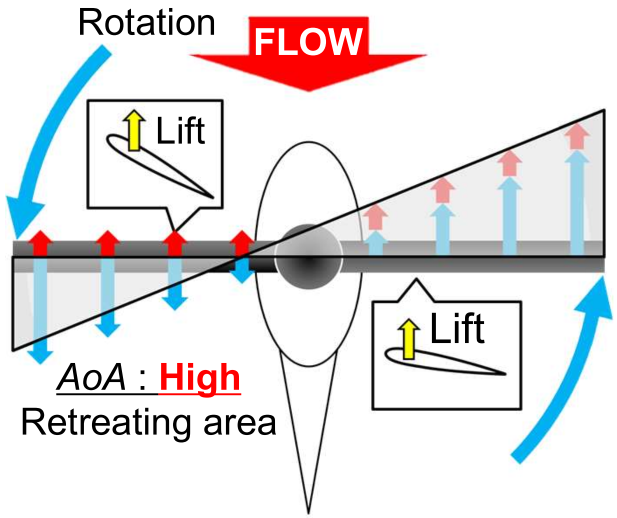
Boeing Higher Education Program Year 2020 Performance Report

Background -Dynamic stall-

- Flow around helicopter blade
  - ✓ Airspeed of blade changes with position
  - ✓ *AoA* changes to keep lift constant
  - ✓ Higher *AoA* is required as speed increases
  - ✓ Dynamic stall occurs in retreating area

➡ **Control of dynamic stall flow is required to increase maximum speed of helicopter**



Velocity distribution of Helicopter blade

- Dynamic stall flow
  - ✓ Flow fields are sensitive to flow condition of pressure, velocity and *AoA*
  - ✓ Different flow is occurred for each rotation

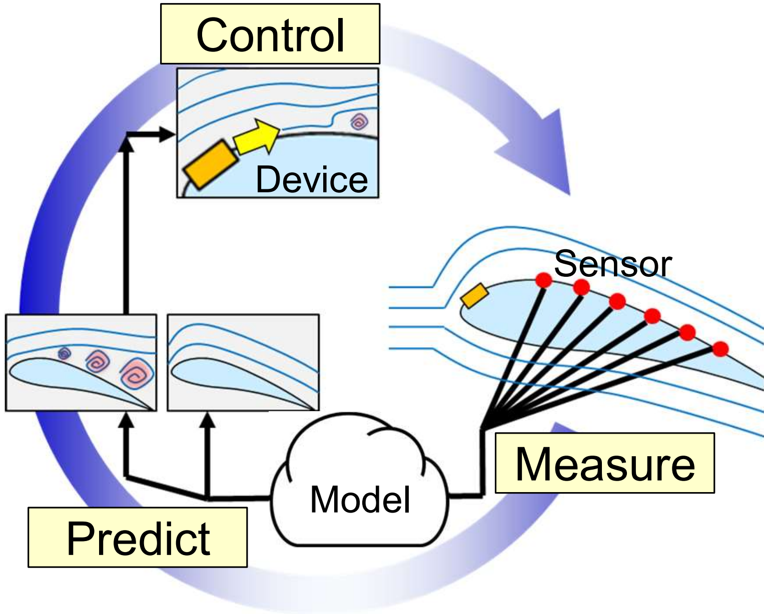
➡ **Optimal feedback control that control separation for each condition is the most effective**

Background & Objective -Optimal feedback control-

☐ Optimal feedback control

- ✓ Active flow control device that has high responsibility
  - Plasma actuator
- ✓ Optimal sensor position around airfoil suitable for understanding flow fields
  - Sparse sensor optimization
- ✓ Reduced-order model that predicts future flow fields instantly
  - Cluster-based reduced-order model (CROM)

… low processing time and high accuracy

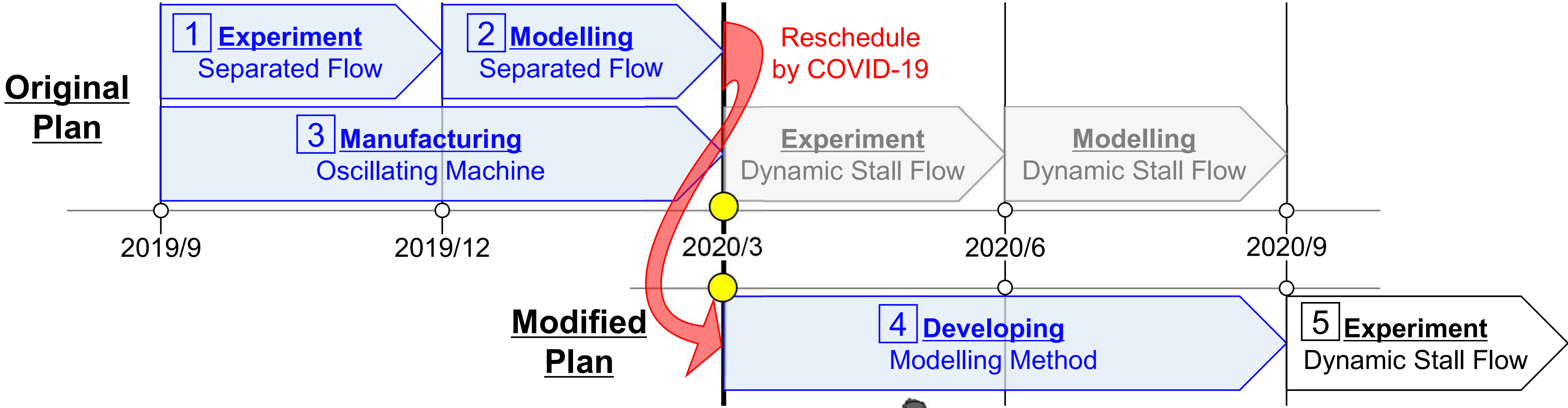


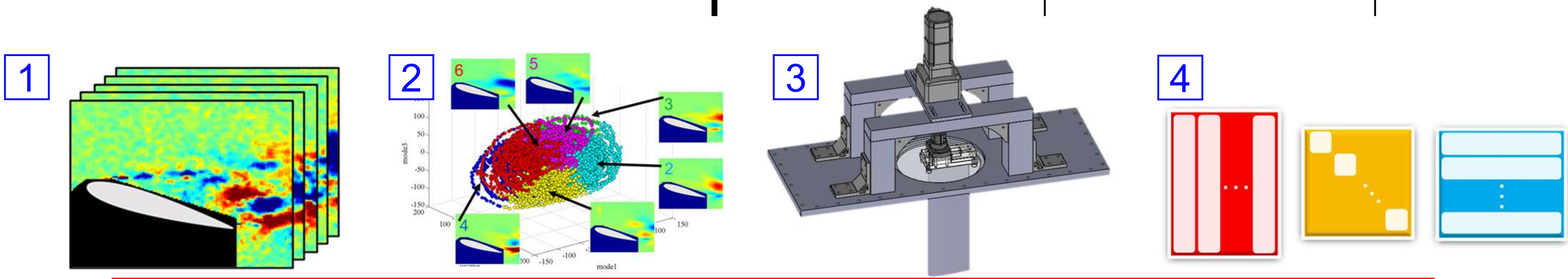
Optimal feedback control of dynamic stall flow

Objective

We constructed CROM of flow fields around airfoil based on experimental data

Project Plan



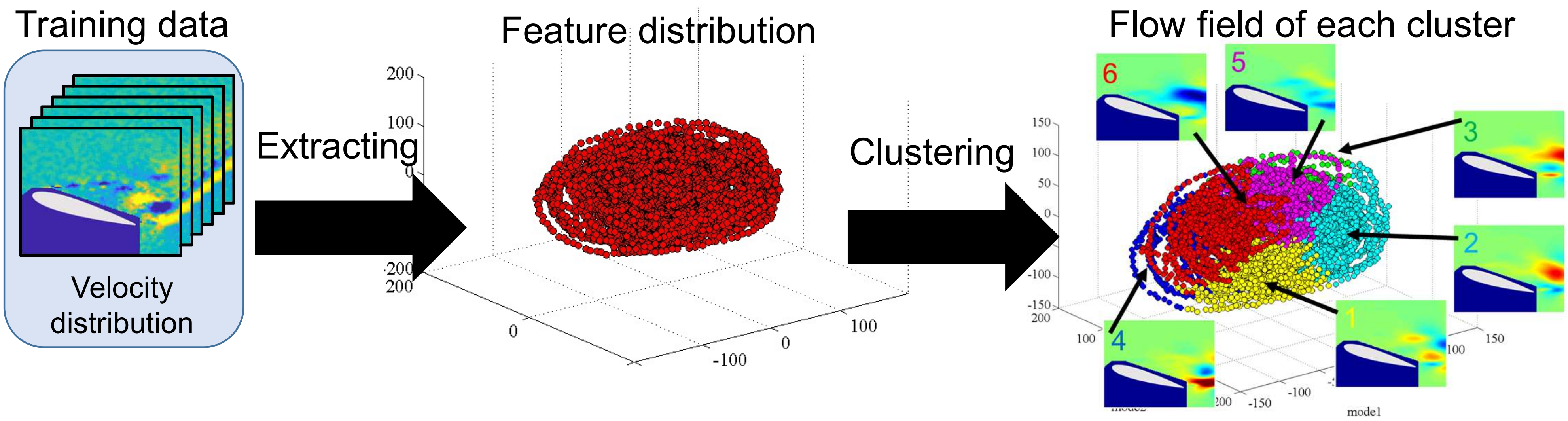


Minimize the bad influence of COVID-19 by rescheduling the project plan

Method -Framework of CROM (1)-

1 2

- Classifying flow fields into cluster
  - ✓ Measure time-resolved velocity distribution of separated flow
  - ✓ Separate training data and test data
  - ✓ Extract feature value from training data by SVD (Singular Value Decomposition)
  - ✓ Classify training data into group of similar flow (clustering) based on feature value



Method -Framework of CROM (2)-

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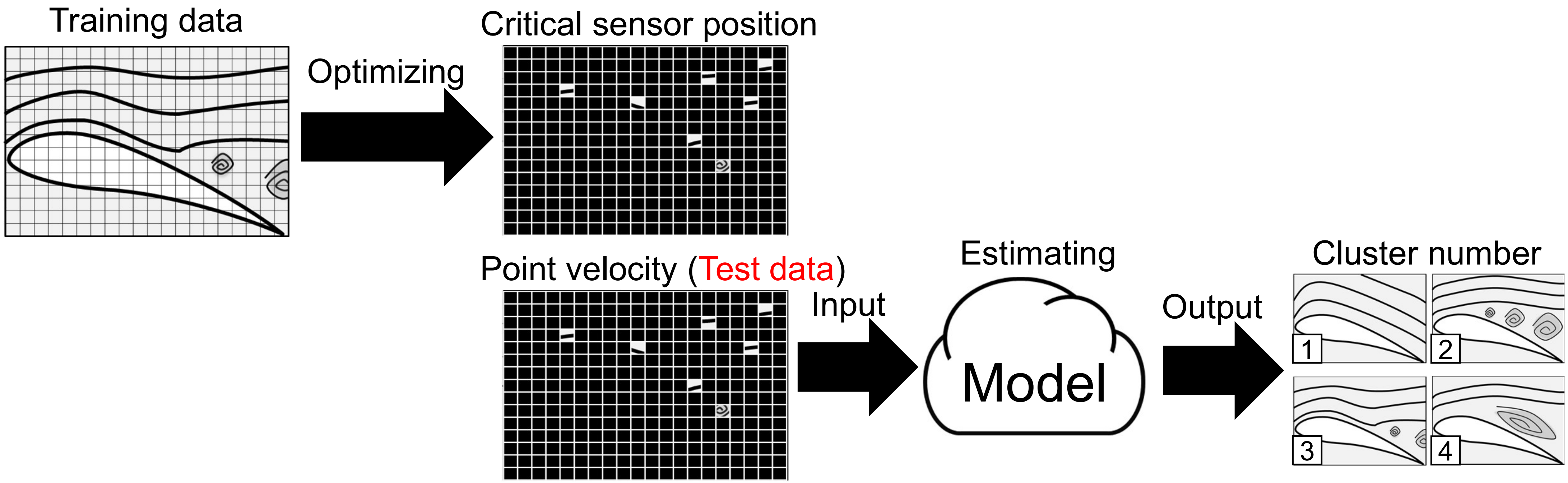
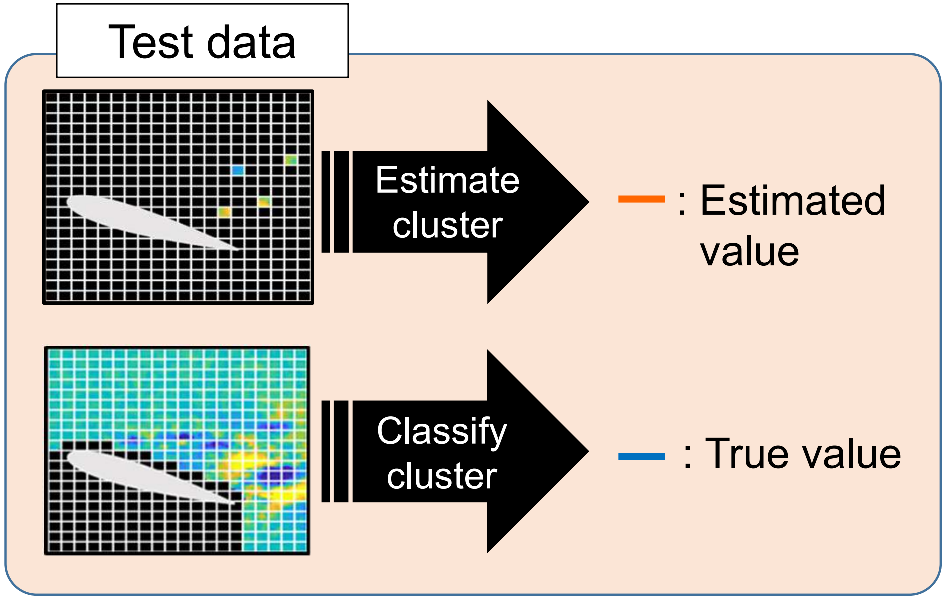
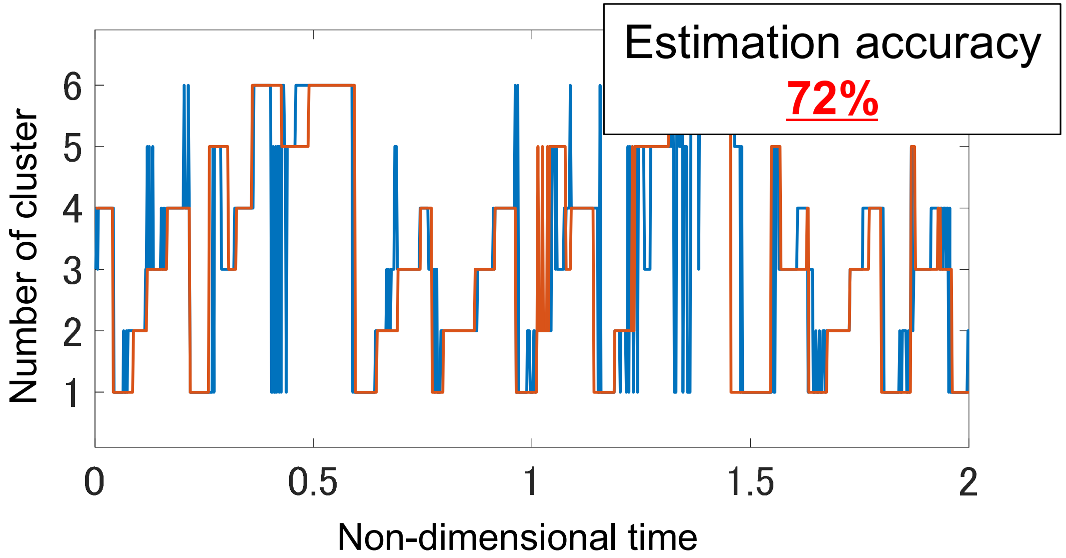
- Estimating cluster number from point velocity (instead of pressure around airfoil)
  - ✓ Find critical sensor position using training data to achieve short processing time
  - ✓ Calculate feature value from some point velocity of test data
  - ✓ Estimate cluster number based on feature distribution



Result -Estimation accuracy of CROM-

1 2





Construct CROM whose estimation accuracy is approximately 72%

However estimation accuracy is not high

Develop model construction method (especially extracting feature value method) to improve estimation accuracy

SVD (Singular Value Decomposition) → DMD (Dynamic Mode Decomposition)

Background -Extracting feature structure method-

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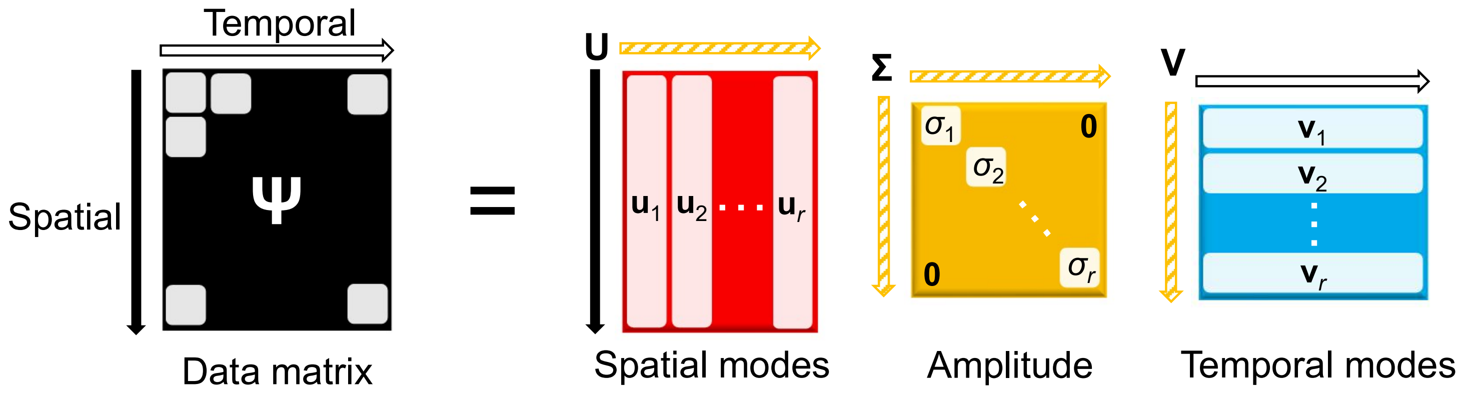
☐ SVD (Singular Value Decomposition) Autonne, 1915



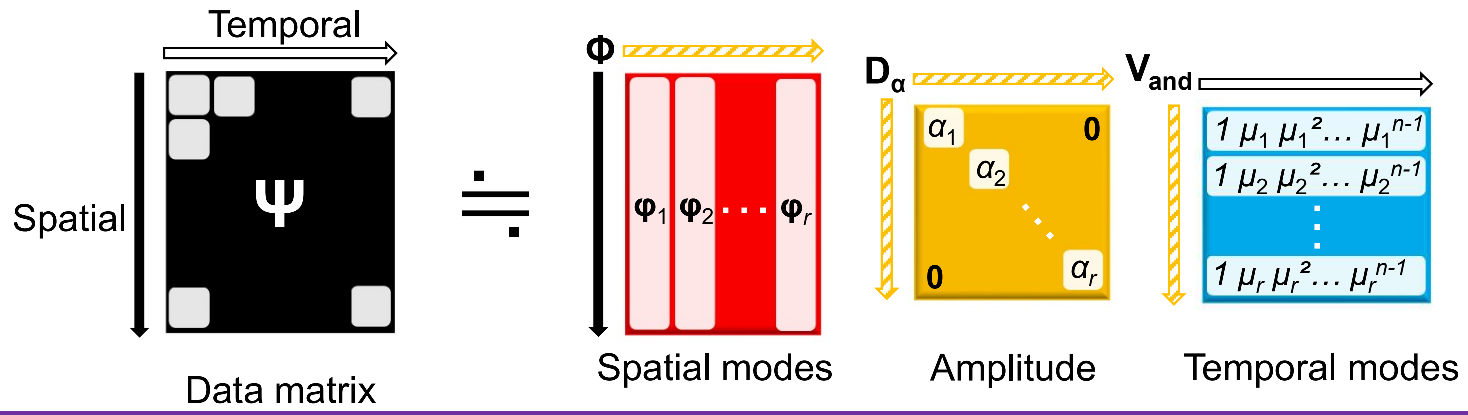
Advantage

- The lowest reconstruction error
- Orthogonal mode
- Easy to choose mode

Disadvantage

- **Periodic phenomena is distributed multiple modes**

☐ DMD (Dynamic Mode Decomposition) Schmid, 2010



Advantage

- **Represent periodic phenomena in a single mode**

Disadvantage

- **Difficult to choose mode**

➡ **Mode selection method**

Previous study

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- DMDsp (sparsity-promoting DMD) … method to find a small number of dominant mode
  - ✓ sDMDsp (system-based DMDsp) Jovanovic, 2014

$$\boxed{\min \left\| \mathbf{\Psi} - \mathbf{\Phi} \mathbf{D}_{\boldsymbol{\alpha}} \mathbf{V}_{\mathrm{and}} \right\|_2^2 + \lambda \left\| \alpha_i \right\|_1}$$

Objective term J (Reconstruction error) — Regularization term (Sum of mode amplitude)

  - ✓ oDMDsp (observation-based DMDsp) Ohmichi, 2019

$$\boxed{\min \left\| \mathbf{\Psi} - \mathbf{\Phi}_s \mathbf{\Phi}_s^{+} \mathbf{\Psi} \right\|_2^2}$$

Objective term J

$\mathbf{\Psi}$: Data
$\mathbf{\Phi}$: Spatial mode
$\boldsymbol{\alpha}$: Amplitude of selected mode
$\mathbf{D}_{\boldsymbol{\alpha}}$: Matrix that diagonal elements are $\boldsymbol{\alpha}$
$\mathbf{V}_{\mathrm{and}}$: Temporal mode
$\mathbf{\Phi}_s$: Selected spatial mode
$\mathbf{\Phi}_s^{+}\mathbf{\Psi}$: Optimal value of selected temporal mode

- Optimization Algorithms
  - ✓ ADMM (Alternating Direction Method of Multipliers) … Adjustment of parameter is required to speed up
  - ✓ Greedy method … Sequential optimization algorithm
  - ✓ FISTA (Fast Iterative Shrinkage Thresholding Algorithm) … Fast algorithm

	ADMM	Greedy method	FISTA
sDMDsp	Jovanovic, 2014	Ohmichi, 2017	×
oDMDsp	×	Ohmichi, 2019	×

Objective

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We apply FISTA to two kinds of DMDsp to realize fast processing time

- Performance evaluation of algorithms
  - ✓ Data : random data matrix
  - ✓ Algorithm : FISTA, ADMM, Greedy method
  - ✓ Metrics of performance evaluation : Processing time, Evaluation term (Reconstruction error)

DMDsp -Two kinds of DMDsp-

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DMDsp

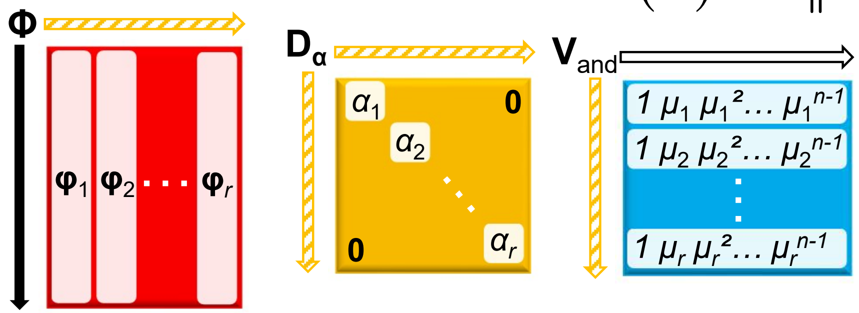
Method to find a small number of dominant modes from data
by minimizing evaluation term and regularization term simultaneously

☐ sDMDsp … system-based method

Evaluation term : $J(\boldsymbol{\alpha}) = \left\| \underline{\boldsymbol{\Psi}} - \underline{\boldsymbol{\Phi}\mathbf{D}_{\alpha}\mathbf{V}_{\mathrm{and}}} \right\|_2^2$

Original | Reconstructed

Optimization problem : $\min J(\boldsymbol{\alpha}) + \lambda \left\| \alpha_i \right\|_1$



Spatial mode | Mode amplitude | Temporal mode

$\mathbf{U}^{-1}\boldsymbol{\Psi}_2 = \mathbf{A}\mathbf{U}^{-1}\boldsymbol{\Psi}_1$
($\mathbf{U}$: spatial SVD mode)

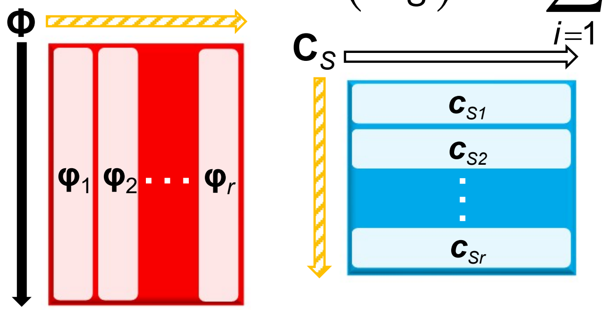
$\boldsymbol{\varphi}$: eigenvector of linear matrix $\mathbf{A}$
$\boldsymbol{\mu}$: eigenvalue of linear matrix $\mathbf{A}$
<u>$\boldsymbol{\alpha}$: mode amplitude (variable)</u>

☐ oDMDsp … observation-based method

Evaluation term : $J(S) = \left\| \underline{\boldsymbol{\Psi}} - \underline{\boldsymbol{\Phi}_S \boldsymbol{\Phi}_S^{+} \boldsymbol{\Psi}} \right\|_2^2$ (Greedy method)

$J(\mathbf{C}_S) = \left\| \underline{\boldsymbol{\Psi}} - \underline{\boldsymbol{\Phi}\mathbf{C}_S} \right\|_2^2$ (FISTA, ADMM)

Optimization problem : $\min J(\mathbf{C}_S) + \lambda \sum_{i=1}^{r} \left\| \mathbf{c}_{Si} \right\|_1$



Spatial mode | Temporal mode

<u>$\boldsymbol{\Phi}_S$: selected spatial modes (variable)</u>
$\boldsymbol{\Phi}_S^{+}\boldsymbol{\Psi}$: optimal value of selected temporal mode
<u>$\mathbf{C}_S$: selected temporal modes (variable)</u>

DMDsp -Optimization algorithm-

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	FISTA	ADMM	Greedy method
Advantage	**Fast convergence**	Easy to apply	**Optimizing number of modes and amplitude**
Disadvantage		**Adjustment of ρ is required to speed up**	
Algorithm	input $\mathbf{x}[0]=\mathbf{0}$ $\mathbf{z}[0]=\mathbf{0}$ $t[0]=1$ while do until convergence $\mathbf{x}[k+1]=S_{\gamma\lambda}\left(\mathbf{z}[k]-\gamma\nabla J(\mathbf{x}[k])\right)$ $t[k+1]=\frac{1+\sqrt{1+4t[k]^2}}{2}$ $\mathbf{z}[k+1]=\mathbf{z}[k]+\frac{t[k]-1}{t[k+1]}(\mathbf{x}[k+1]-\mathbf{x}[k])$ $k=k+1$ output $\mathbf{z}$ $S_{\gamma\lambda}$: Soft thresholding operator γ : Convergence acceleration rate (Optimal)	input $\mathbf{z}[0]=\mathbf{0}$ $\mathbf{v}[0]=\mathbf{0}$ while do until convergence $\mathbf{x}[k+1]=\underset{\mathbf{x}}{\arg\min}\,J(\mathbf{x})+\frac{\rho\left\Vert\mathbf{v}[k]+\mathbf{x}-\mathbf{z}[k]\right\Vert_2^2}{2}$ $z_i[k+1]=S_{\frac{\gamma}{\rho}}\left(x_i[k+1]+\frac{v_i[k]}{\rho}\right)$ $\mathbf{v}[k+1]=\mathbf{v}[k]+\rho\left(\mathbf{x}[k+1]-\mathbf{z}[k+1]\right)$ $k=k+1$ output $\mathbf{z}$ ρ : Convergence acceleration rate (minimum)	input K while do $r<K$ $j_0=\arg\min J\left(S^r\cup j\right), j\notin S^r$ $S^{r+1}=S^r\cup j_0$ $r=r+1$ Output S K : Number of modes

Data matrix

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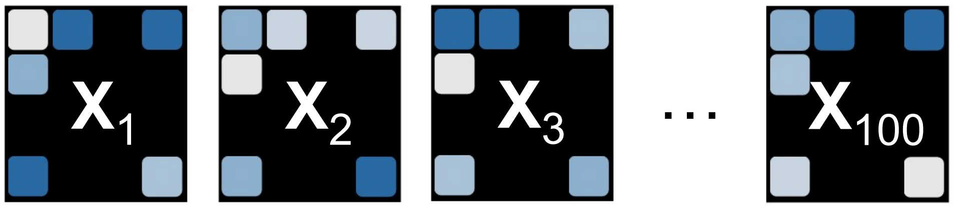
- **Random data matrix** … evaluate algorithm performance independent to data
  - ✓ Data size (column) : 1000
  - ✓ Data size (row) : 100
  - ✓ Value : Random numbers according to standard normal distribution (Average : 0 Dispersion : 1)
  - ✓ Number of data : 100



Result -Comparison of sDMDsp algorithms-

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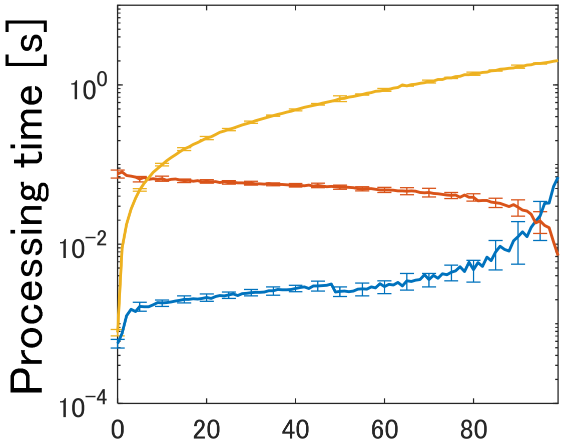
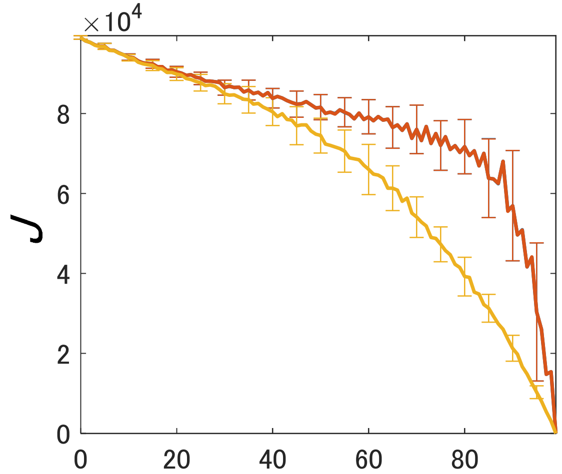


Comparison of processing time



Comparison of evaluation term

- FISTA
- ADMM (conventional)
- Greedy method (conventional)

Evaluation term : $J(\boldsymbol{\alpha}) = \left\| \underbrace{\boldsymbol{\Psi}}_{\text{Original}} - \underbrace{\boldsymbol{\Phi} \mathbf{D}_{\boldsymbol{\alpha}} \mathbf{V}_{\text{and}}}_{\text{Reconstructed}} \right\|_2^2$

(Reconstruction error)

Processing time : FISTA < ADMM << Greedy method

Evaluation term (Few modes) : Greedy method ≦ FISTA = ADMM

Evaluation term (Many modes) : Greedy method < FISTA = ADMM

(FISTA and ADMM choose same modes)

sDMDsp applying FISTA realizes the fastest processing time in three algorithms

Result -Comparison of oDMDsp algorithms-

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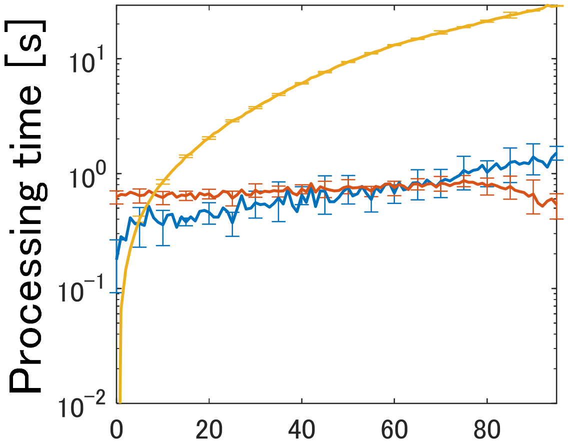
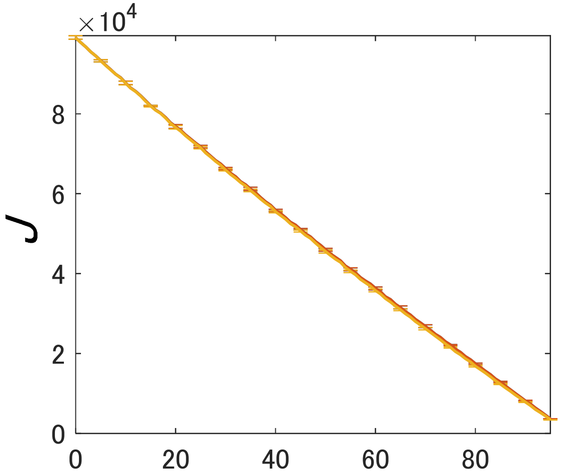


Comparison of processing time



Comparison of evaluation term

FISTA
ADMM
Greedy method (conventional)

Evaluation term : (Reconstruction error)

Greedy method

$$J(S) = \left\| \underline{\boldsymbol{\Psi}} - \underline{\boldsymbol{\Phi}_S \boldsymbol{\Phi}_S^{+} \boldsymbol{\Psi}} \right\|_2^2$$

FISTA, ADMM

$$J(\mathbf{C}_S) = \left\| \underline{\boldsymbol{\Psi}} - \underline{\boldsymbol{\Phi} \mathbf{C}_S} \right\|_2^2$$

Original | Reconstructed

Processing time (Few modes) : FISTA < ADMM << Greedy method

Processing time (Many modes) : ADMM ≦ FISTA << Greedy method

Evaluation term : Greedy method ≒ FISTA = ADMM

(FISTA and ADMM choose same modes)

oDMDsp applying FISTA and ADMM realize faster processing time than Greedy method

Conclusions

We constructed CROM whose estimation accuracy was 72% of flow fields around airfoil based on experimental data

We applied FISTA to two kinds of DMDsp and realized faster processing time than DMDsp applying conventional algorithms

◻ Evaluation of algorithm performance

✓ sDMDsp

- Processing time : FISTA < ADMM << Greedy method
- Evaluation term (Few modes) : Greedy method ≦ FISTA ＝ ADMM
- Evaluation term (Many modes) : Greedy method < FISTA ＝ ADMM

✓ oDMDsp

- Processing time : FISTA < ADMM << Greedy method
- Evaluation term (Few modes) : Greedy method ≦ FISTA ＝ ADMM
- Evaluation term (Many modes) : Greedy method < FISTA ＝ ADMM