Shock Wave Reflections in Axisymmetric Flow

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Abstract. The paper summarizes results of previous theoretical studies on the shock wave reflection from the axis of symmetry and illustrates them using high-resolution numerical modelling based on the Euler-equations/perfect-gas assumption. It is confirmed, for both steady and moving shock waves, that only a Mach reflection can exist at the axis of symmetry, although the size of its Mach disk may be very small. Numerical convergence studies support the physical feasibility of highly curved Mach disks. However, it is shown that in some cases they are really numerical artefacts.

1 Introduction

Shock wave interaction with the axis of symmetry is a basic problem which has been studied at least since the 1940's. Possible practical applications include explosions and implosions, axisymmetric nozzle flows, and scramjet engine inlets. The problem is rather well elaborated theoretically. However, the results are scattered over a number of small circulation publications. Moreover, the theoretical conclusion on the impossibility of regular reflections from the axis of symmetry looks, at first glance, somewhat puzzling in view of experimental and numerical findings with seemingly regular shock reflections. As a result, in textbooks and monographs the issue is either missed altogether, or presented incompletely/unclearly, or even interpreted erroneously. The present paper is intended as a summary of theoretical works on the subject, illustrated by high-resolution numerical simulations. The Euler-equations/perfect-gas model embodied in the locally adaptive unstructured code [1] has been adopted because it is the simplest possible one to begin with and at the same time all theoretical studies are based upon it.

2 Steady Reflections

In 1948 Courant and Friedrichs [2] put forward their famous theoretical conclusion, based on velocity hodograph considerations, that, in steady flow, an incident conical shock, followed by a reflected conical shock with an intervening conical flowfield, cannot exist at the axis of symmetry and that a Mach reflection would have to occur. However, experiments and numerical studies, particularly with weak converging shocks, have produced images of shock patterns, in the

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vicinity of the axis of symmetry, with seemingly regular shock reflections. The obvious contradiction has been attributed, first of all by Courant and Friedrichs themselves, to the existence of a Mach disk that is "too small to be seen".

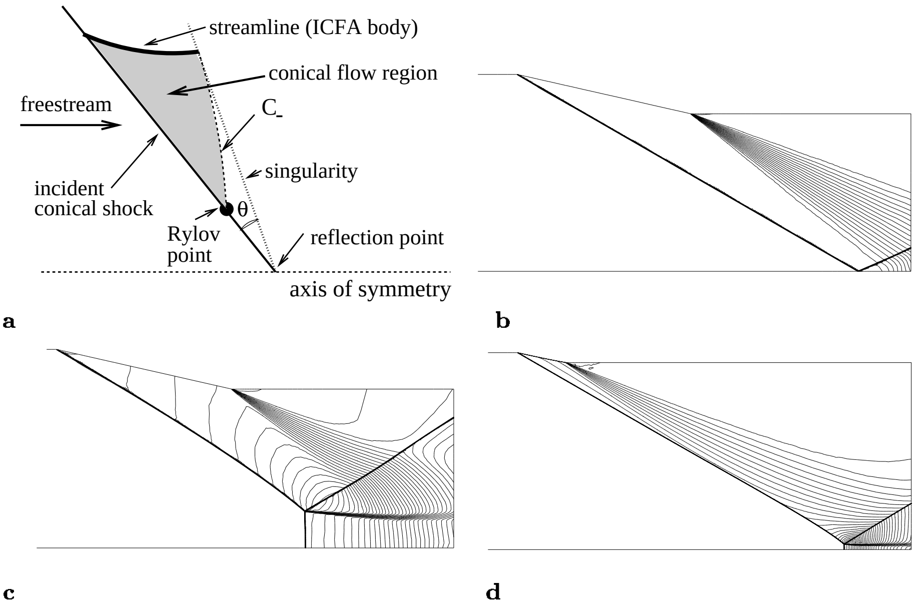


Fig. 1. Steady reflections in internal flows. (**a**) Schematics of the ICFA flow. (**b-d**) Numerical results (density contours) for the freestream Mach number $M = 3$ and the incident shock angle $\alpha = 30°$ ($\gamma = 1.4$): (**b**) Plane flow, the wedge angle $\beta = 12.7735°$; (**c**) Axisymmetrical flow, the geometry is the same as for the case (**b**); (**d**) Axisymmetrical flow with the ICFA ring body designed to produce the same incident shock angle 30°

Taking a different point of view, it was suggested in [3–5] to consider a conical shock impinging on the axis (see Fig. 1a) and to try constructing a conical (i.e. depending only on the polar angle θ) flow downstream of it. Such a flow can be formally obtained right down to a singular line (Fig. 1a) at which derivatives of some physical variables with respect to the polar angle tend to infinity and further conical flow is impossible. Any streamline of the conical, self-similar flow (the Internal Conical Flow A – ICFA, according to [4]) can be declared to be a body surface – the ICFA body. However detailed analysis of the flowfield [5] reveals that conical flow is physically feasible only within the area shaded in Fig. 1a and bounded by the incident conical shock, the ICFA body and the C_- characteristic arriving at the trailing edge of the ICFA body and being tangential there to the singular line. The C_- characteristic intersects the incident shock at a point which is called the Rylov point in the present paper. Downstream of the Rylov point, the ICFA body is no longer able to influence and straighten the incident shock and, due to converging geometry, the shock is amplified and steepened towards the axis so that eventually a Mach reflection takes place there.

To verify the above findings numerically, we consider a combination of shock angle and freestream Mach number which puts a triple-shock solution beyond the von Neumann point, where a regular reflection would occur for the plane flow case (Fig. 1b). In axisymmetric flow this combination of Mach number and shock angle is produced at the leading edge of an axisymmetric wedge-ring. However, in progressing towards the axis of symmetry, the shock steepens to the von Neumann point and Mach reflection ensues (Fig. 1c). By replacing the wedge-ring with the ICFA-profiled ring body we are able to sustain the straight conical shock from the leading edge till the radius equal to $\sim 0.4R_{le}$ (the Rylov point), where R_{le} is the leading edge radius (Fig. 1d). The value agrees quite well with the theoretical prediction ($0.386R_{le}$). Numerical simulations using the Euler code as well as the method of characteristics show that beyond that point it is impossible to keep the shock straight. Even applying the highest possible rate of downstream expansion with a centered Prandtl-Meyer fan at the trailing edge of the ICFA surface, we cannot prevent the increase of shock intensity and incident angle and consequent Mach reflection. The relatively high incident shock angle 30° was chosen for the sake of faster computations and easier presentation. In [6] we have computed the ICFA flow with the freestream Mach number $M = 8.33$ and shock incident angle as low as $\alpha = 9.5°$ where the straight shock portion extended to $0.163R_{le}$ and then exhibited irregular reflection even for such a low initial incident angle. In this case, the size of the Mach disk was indeed very small ($\sim 1.2 \cdot 10^{-4} R_{le}$). This seems reasonable since a longer relative distance is needed for the initially low incident angle to exceed the von Neumann value.

3 Unsteady Reflections

Let us first consider the case of an axisymmetrical shock wave (for instance, a toroidal shock wave, see Fig. 2) approaching the axis with seemingly zero angle of incidence, where one might most likely expect a regular reflection pattern. Sokolov, in [7] suggested a gasdynamical mechanism which always leads to Mach reflection at the axis of symmetry during an unsteady reflection process. The explanation suggests the amplification and acceleration of a shock wave, approaching the axis of symmetry. As a result, shock wave front segments closer to the axis are more intense and move faster than those away from it, resulting in a steepening of the shock's incident angle. At any given finite distance from the cumulation point the shock angle exceeds the transition angle. The curvatures of the incident shock at the triple point and far away are of different sign. A point of zero curvature exists somewhere in between: Fig. 2 shows that in grossly exaggerated manner. In reality, the variation of curvature takes place gradually over a long axial distance, so that it is difficult to resolve it numerically. The Mach disk radius r_m vs. the axial coordinate z is given by

$$\frac{r_m}{R_0} = \left(\frac{z}{R_0}\right)^n \frac{1}{(\sin\alpha)^n \cos\alpha} \ ,$$

where R_o is a constant having the dimension of length; α – the angle of incidence. The power coefficient n is ≈ 5.1 for $\gamma = 1.4$ – that is why the numerical confirmation via a grid convergence study requires successive grid refinement in about 32(!) times (not in two times as it is usually done). Our previous attempt ([8]) was incomplete because of insufficient resolution of structured non-adaptive numerical codes on hand at that time, although it clearly demonstrated that a finer mesh leads to earlier detection of the Mach disk.

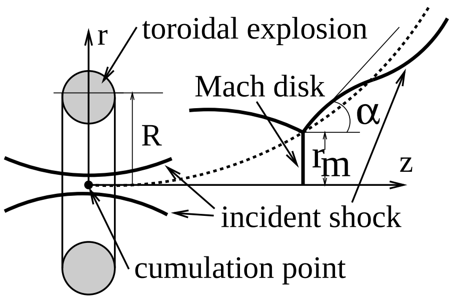


Fig. 2. Schematics of the toroidal shock wave reflection from the axis of symmetry (z-axis). A cross-section of the initial energy deposition volume as well as shock fronts before and after reflection are shown. Dashed line depicts the triple point trajectory

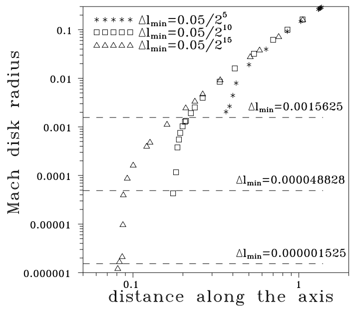


Fig. 3. Toroidal shock wave reflection from the axis of symmetry: Mach disk radius vs. the distance along the axis from the cumulation point for different minimum grid spacings. All quantities are non-dimensionalized with the large toroidal radius R

The present numerical simulations (Figs. 3,4) were performed with the background grid step $\approx 0.05R$ (R is the large radius of the initial toroidal pressurized volume while its small radius is r_s) and 5, 10, or 15 levels of grid refinement applied near the axis of symmetry (as soon as Mach disk radius became larger than 50 cells of a certain refinement level, the level was removed). Figure 4a shows the amplification and acceleration of the incident shock near the axis just before the cumulation moment. Due to inherently finite grid resolution of shock-capturing techniques, on any grid we have seemingly regular reflection at early stages (Fig. 4b). However, the substantial increase of the incident angle close to the symmetry axis and the above-mentioned variation of curvature is very clearly resolved in Fig. 4b, with the zero curvature point being far beyond the right margin of the figure. Eventually Mach reflection appears (Fig. 4c). Figure 3 proves that to resolve the Mach disk two times closer to the cumulation point, about 32 times finer mesh is needed. Once having appeared, the Mach disk initially grows very fast until it reaches a proper (grid-independent) value corresponding to the particular distance.

The above unsteady results deal with an axysymmetric shock wave impinging on the symmetry axis at seemingly zero incident angle (quite frequent case in

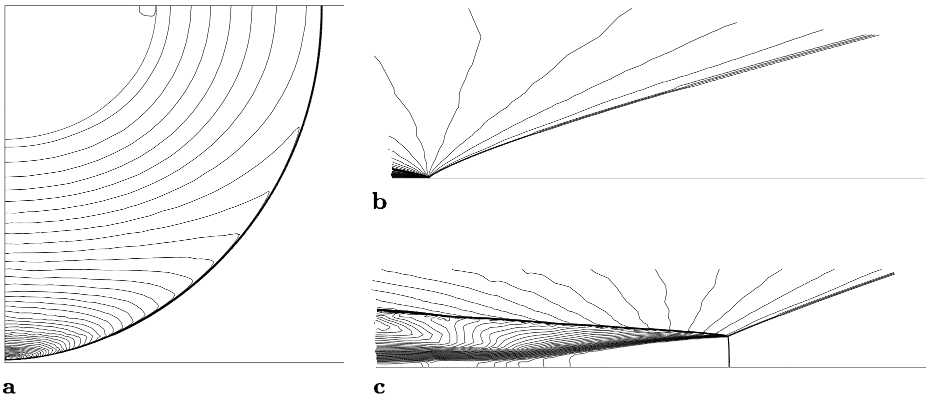


Fig. 4. Toroidal shock reflection from the axis of symmetry (pressure (**a**) and density (**b**, **c**) contours in cross-sections are shown): (**a**) the computational domain is bounded by the axis of symmetry (lower horizontal line), the plane of symmetry (vertical line) and a cylindrical wall (upper horizontal line), in order not to consider outgoing waves; a moment prior to cumulation; (**b**) a typical regular reflection pattern at an early stage of reflection; (**c**) a typical Mach reflection pattern developed later. Initial parameters of the explosion: $p_2/p_1 = 50$, $\rho_2/\rho_1 = 10$, $r_s/R = 0.2$

practice). If we launch a shock towards the axis at a finite angle (Fig. 5a), the physical situation is more transparent. Different segments of the shock move with different speed, so that the incident angle increases, exceeding the transition value (see Fig. 5b, which is an unsteady analog of Fig. 1c), resulting immediately in Mach reflection (Fig. 5c,d).

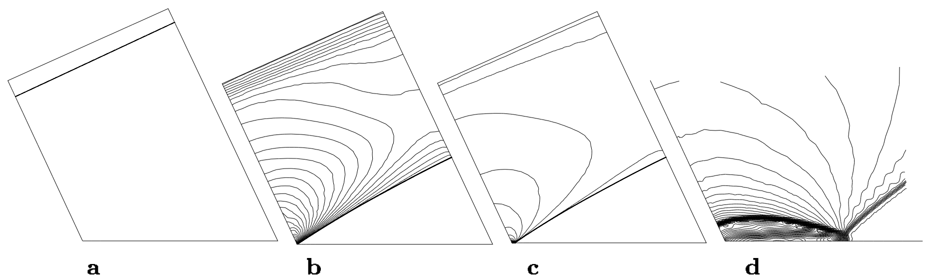
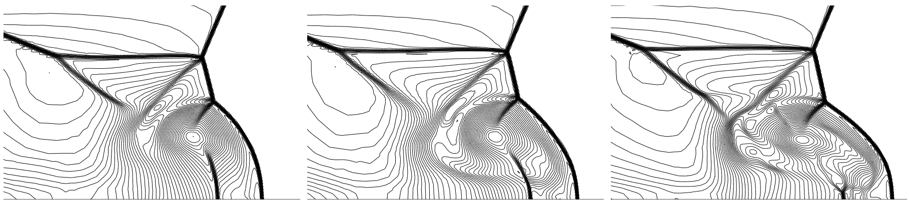


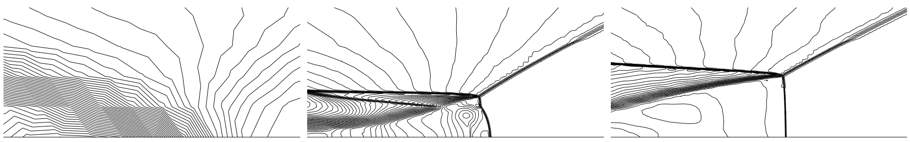
Fig. 5. The reflection of an initially plane (conical) axisymmetrical shock front ($M = 3$, $\alpha = 25°$) from the axis of symmetry (lower horizontal line); density contours are shown: (**a**) initial condition, inclined lines are conical walls; a moment just before (**b**) and after (**c**) reflection; (**d**) enlargement of image (**c**) near the reflection point

During the last decade many researchers reported that in both steady and unsteady flows the Mach reflection patterns at the axis of symmetry can be of complex structure, with a highly curved Mach disk, additional triple point,



a b c

Fig. 6. Toroidal shock reflection from the axis of symmetry: shock pattern for $0.83 \leq z/R \leq 1.15$, $0 \leq r/R \leq 0.21$ obtained with 5 (**a**), 10 (**b**) and 15 (**c**) levels of grid refinement near the cumulation point (see also Fig. 3)



a b c

Fig. 7. Toroidal shock reflection from the axis of symmetry: shock pattern for $0.19874 \leq z/R \leq 0.23164$, $0 \leq r/R \leq 0.065$ obtained with 5 (**a**), 10 (**b**) and 15 (**c**) levels of grid refinement near the cumulation point (see also Fig. 3)

axial jet and toroidal vortex (Fig. 6). However, experimental confirmation has not been achieved so far. In our review, [9], we even considered the possibility of the phenomena being numerical artefacts. In a more recent study [10] the problem was investigated in a self-similar formulation; the domains of existence for different types of the bulged Mach configuration were established. The very self-similarity of the phenomenon supports its possible physical existence. The present computation with 15 refinement levels shows that the phenomenon is indeed unlikely related to some numerical effects near the cumulation point: the Mach disk emerges as a plane one (Fig. 4c) and remains plane for a rather long time while growing a few hundreds of finest cells heights; only later, under a certain combination of incident Mach number and angle, which seems to be close to self-similar predictions in [10], does the bulging disk appear. Figure 6 shows the results of grid convergence study: the overall structure is very much the same, so that it seems to be physically feasible. However, finer meshes result in more protruding Mach disks, not exhibiting grid convergence for this particular feature. This suggests stronger axial jets which may result from deficiencies of the Euler model not taking into account dissipative and real gas effects which could be important at high pressures and temperatures near the cumulation point and the axis. Figure 7 corresponds to an area closer to the cumulation point, for which the intermediate mesh produces a curved Mach disk (Fig 7b) disappearing upon further grid refinement (Fig. 7c). We note that Fig. 7b corresponds to the stage of fast growth of Mach disk, which is as aphysical as the preceding stage of ‘regular’ reflection (Fig. 3).

4 Conclusion

The Euler-equations/perfect-gas model for both steady and unsteady flow predicts an irregular Mach reflection configuration for shock waves reflecting from the axis of symmetry. In all cases the incident wave angle steepens toward the axis so as to exceed the von Neumann angle at the triple point. The Mach disk may indeed be very small as compared to the characteristic scale of the problem under study and, thus, invisible on experimental photos or images resulting from low resolution numerical computations.

Although the employed model does not contain any length scales, it is worth to provide some estimates for a laboratory-scale experiment. Assuming that $R = 10$ cm for the toroidal explosion, the Mach disk radius, at $z < 3$ cm, is less than 1 mm (a visibility threshold for optical visualizations). It is less than 1 μm for $z < 0.8$ cm. At these distances, when the radius becomes comparable with the shock thickness, the continuum model is not valid any longer.

Caution should be exercised when interpreting results with a highly curved Mach disk because the Euler model may not provide quantitative agreement with possible future experimental findings, since it predicts unphysically high, grid-dependent pressure and temperature values near the cumulation point and axis of symmetry and, as a consequence, more intense axial jets and vortexes. Moreover, it has been shown that in some cases the curved Mach disks disappear when finer mesh is used.

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