Artificial Wind Schemes as a Simple and Efficient Tool to Simulate Diverse Shock Wave Flows

Eugene Timofeev[1], Igor' Sokolov[2*], Jun-ichi Sakai[2], Peter Voinovich[1**], Nobuo Nagayasu[1,3], and Kazuyoshi Takayama[1]

[1] Shock Wave Research Center, Institute of Fluid Science, Tohoku University, 2-1-1 Katahira, Aoba-ku, Sendai 980-8577, Japan
[2] Laboratory for Plasma Astrophysics, Faculty of Engineering, Toyama University, 3190 Gofuku, Toyama 930-8555, Japan
[3] Chugoku Kayaku Co. Ltd., 5-1-1 Etajuma-cho, Aki-gun, Hiroshima 737-21, Japan

Abstract. The paper outlines the Artificial Wind schemes – a recently proposed class of upwind schemes, the main advantages of which in comparison with traditional approaches are their simplicity and ability to treat a variety of hydrodynamics models without or with minor generalizations. The emphasis is given to applications of the schemes to different shock wave problems.

1 Introduction

Nowadays upwind shock-capturing schemes are widely used to compute flows with shock waves. The general idea of upwinding implies that the numerical flux via a face between two adjacent grid cells should be decomposed into the sum of perturbations propagating across the face in one direction and those propagating in the opposite direction, and numerical differences should be taken accordingly. This approach, based on reasonable physical arguments, usually leads to high quality numerical results.

At present, shock wave researchers are typically interested in basic and applied problems which involve much more sophisticated physico-mathematical models as compared to the simplest Euler-equations/perfect-gas approach. The application of upwind schemes to such complex modeling, although quite satisfactory from the point of view of accuracy and robustness, reveals at the same time their unpleasant shortcomings.

First of all, for nonlinear equations, such as the hydrodynamic ones, the practical construction of upwind numerical schemes (i.e. procedures to separate the perturbations) is typically accompanied by a lot of technical difficulties to be overcome, thus resulting in sophisticated algorithms. It may be necessary to find, for instance, the Roe matrix with the total set of its eigenvectors and eigenvalues, to calculate exact or approximate solutions of the Riemann problem, or

* Present address: The University of Michigan, Ann Arbor, MI 48109-2143, USA
** Permanent address: Soft-Impact Ltd., P.O. Box 33, 194156 St. Petersburg, Russia

to apply flux splitting procedures. Secondly, these tasks become even more sophisticated for complex physical problems in which, for instance, the effects of real fluids with complex equations of state, equilibrium/nonequilibrium chemistry and thermodynamics, magnetohydrodynamics, special relativity etc. should be taken into account. In some cases formulas for explicit calculations occupy literally several journal pages requiring tedious programming and reducing computational efficiency. Finally, each particular physical model may require new generalization of an upwind scheme and, thus, very substantial modifications of existing codes.

That is why some time ago we embarked on a quest to develop a simple upwind numerical technique which could be easily, without or with minor generalization efforts, applied to a variety of hydrodynamic problems involving quite different and complex physico-mathematical models, which we are facing.

The aim of the present paper is to introduce readers to the main ideas and principles underlying the new approach for which the term "Artificial Wind" was coined. We refer to [1] for detailed mathematical derivations, proofs, interpretations, and tests while giving here more details and illustrations on shock wave research applications.

2 Artificial Wind

Our main idea for the design of upwind schemes is to avoid entirely the splitting of perturbations, the key and most cumbersome part of any upwind scheme, by making all of them propagating in the same direction depending on our choice and, thus, eradicating at once all complexities of upwind flux construction. The very possibility of this technique has its roots in the Galilean invariance property of hydrodynamic equations, which stipulates that they retain the same form in all frames of reference moving with a constant speed relatively each other, and that all such systems of coordinates are equally qualified for solving the equations.

This can be practically achieved by various means:

- choosing the frame of reference which moves with a velocity D with respect to the original frame of reference in such way that the flow under study would be supersonic in the new frame;
- moving control volume faces during a time step with a velocity D ensuring supersonic flow with respect to each face;
- considering a general spatial-temporal transformation $t' = t$, $x' = x - Dt$ for a hyperbolic conservation law $\partial \mathbf{U}/\partial t + \partial \mathbf{F}/\partial x = 0$ which leads to the conversion of all eigenvalues of the Jacobian $\mathbf{A}' = \mathbf{A} - D\mathbf{E}$ into either positive or negative values.

All the ways are physically equivalent (they result from the Galilean invariance) and interrelated. They all introduce an additional velocity D which is called the Artificial Wind velocity to emphasize that its value is a matter of our choice and that it is introduced to facilitate upwinding.

There are different ways to choose the values of Artificial Wind velocities. They can be chosen to be the same within the whole computational domain (for a given time step) – so-called Global Artificial Wind. They may be assigned separately for each control volume face resulting in the Local Artificial Wind schemes. Alternatively, they could be determined separately for each intermediate state introduced between the left and right hydrodynamic states at a control volume face (Differential Artificial Wind). Simple theoretical considerations and numerical tests suggest that the AW velocities are to be kept as low as possible ensuring lowest numerical dissipation (and at the same time being most favorable for stability). From this point of view the Differential Artifical Wind scheme is the best option.

The Differential Artificial Wind scheme possesses some important properties. First, its flux for a non-linear hyperbolic scalar conservation law with a convex flux is identical to that of the Godunov scheme. Second, it can be interpreted as the Godunov scheme using a modified initial distribution of gasdynamic parameters within control volumes. Third, the scheme satisfies the entropy non-decreasing condition. It is essential that its construction procedure requires only the knowledge of the maximum and minumum eigenvalues of the hyperbolic system of conservation laws under consideration and does not involve any additional assumptions, thus being very general.

The above-mentioned intermediate states may be introduced as a continuous (for instance, linear) distribution. However, we may also include a physical discontinuity into it. From practical point of view it is important to incorporate a contact discontinuity, thus essentially improving their resolution. To make the choice of internal contact discontinuity unique, we impose the requirement for the resulting scheme to be entropy-nondecreasing.

At the final stage of derivation, we should specify the equation of state. The Differential Artificial Wind scheme with Contact Discontituity for the Euler equations ($\mathbf{U} = (\rho, \rho u, \rho E)$) and perfect gas case ($\rho E = p/(\gamma - 1) + 0.5\rho u^2$; $c = \sqrt{\gamma p/\rho}$) can be written as:

$$\mathbf{F}_{i-1/2} = \begin{cases} \mathbf{F}(\mathbf{U}_R) - \dfrac{d}{1-\xi^*}(\mathbf{U}_i - \mathbf{U}_R), & \text{if } u_C - D^C \le 0; \\ \mathbf{F}(\mathbf{U}_L) - \dfrac{d}{\xi^*}(\mathbf{U}_L - \mathbf{U}_{i-1}), & \text{if } u_C - D^C \ge 0; \end{cases} \tag{1}$$

$$D^C = \frac{d \cdot (\xi^c - \xi^*)}{\xi^* \cdot (1 - \xi^*)}; \tag{2}$$

$$u_C = u_L = u_R = \frac{\rho_{i-1} u_{i-1} \cdot (1 - \xi^*) + \rho_i u_i \cdot \xi^*}{\rho_{i-1} \cdot (1 - \xi^*) + \rho_i \cdot \xi^*}; \tag{3}$$

$$p_L = p_R = p_{i-1} \cdot (1 - \xi^*) + p_i \cdot \xi^* + \delta Q_k (\gamma - 1); \tag{4}$$

$$\delta Q_k = \frac{1}{2} \cdot \frac{\rho_{i-1}\rho_i (1 - \xi^*)\xi^* (u_i - u_{i-1})^2}{\rho_{i-1} \cdot (1 - \xi^*) + \rho_i \cdot \xi^*}; \tag{5}$$

$$\rho_L = \rho_{i-1} \cdot \frac{1 - \xi^*}{1 - \xi^c}; \tag{6}$$

$$\rho_R = \rho_i \cdot \frac{\xi^*}{\xi^c}; \tag{7}$$

$$\xi^c = \frac{\xi^*}{\xi^* + (1-\xi^*)(p_{i-1}/p_i)^{1/\gamma}}; \tag{8}$$

$$d = \int_0^{\xi^*} D^R(\xi)\mathrm{d}\xi = -\int_{\xi^*}^1 D^L(\xi)\mathrm{d}\xi; \tag{9}$$

$$D^R(\xi) = \max\{0, u(\xi) + c(\xi)\}; \tag{10}$$

$$D^L(\xi) = \min\{0, u(\xi) - c(\xi)\}, \tag{11}$$

where D^L, D^R and D^C are the Artificial Wind velocities. The weight coefficient ξ is determined from (9) using a simple iteration procedure converging in 2-3 iterations:

$$\widetilde{D}^L(\xi^{*(k)}) = 0.5(\min\{u_i - c_i, 0\} + \min\{u(\xi^{*(k)}) - c(\xi^{*(k)}), 0\}), \tag{12}$$

$$\widetilde{D}^R(\xi^{*(k)}) = 0.5(\max\{u_{i-1} + c_{i-1}, 0\} + \max\{u(\xi^{*(k)}) + c(\xi^{*(k)}), 0\}), \tag{13}$$

$$\xi^{*(k+1)} = -\widetilde{D}^L(\xi^{*(k)})/[\widetilde{D}^R(\xi^{*(k)}) - \widetilde{D}^L(\xi^{*(k)})] \ . \tag{14}$$

The first approximation $\xi^{*(1)}$ is taken from the Local Artificial Wind formula:

$$\xi^{*(1)} = -D^L_{i-1/2}/(D^R_{i-1/2} - D^L_{i-1/2}); \tag{15}$$

$$D^R_{i-1/2} = \max\{u_{i-1} + c_{i-1}, u_i + c_i, 0\}; \tag{16}$$

$$D^L_{i-1/2} = \min\{u_{i-1} - c_{i-1}, u_i - c_i, 0\}. \tag{17}$$

The diffusion coefficient may be then calculated as

$$d = \max\{\xi^{*(k+1)}\widetilde{D}^R(\xi^{*(k+1)}), -(1-\xi^{*(k+1)})\widetilde{D}^L(\xi^{*(k+1)})\} \ . \tag{18}$$

The second order in space and time can be achieved in a variety of ways ([2]). In our codes we employ the Rodionov [3] (or MUSCL-Hancock [2]) predictor-corrector approach with a TVD limiter.

3 Applications

First, we mention here the studies of shock wave propagation in water assuming that the Tait equation of state is valid and employing the locally adaptive unstructured 2D and 3D codes [4]. In fact, almost the same formulas as those for perfect gases can be used in this case. The only difference is that the expressions for the internal energy and the speed of sound should be slightly modified as follows from the Tait equation.

Two problems were analyzed. The first one is the long distance propagation of a shock wave produced in the ocean by an impact of a celestial body [5], involving shock wave interactions with the ocean free surface and deep ocean sound channel. The second problem is the interaction of underwater shock waves induced by an explosion of mild detonating fuses: two submerged segments of mild detonation fuse are attached to each other at a certain angle and ignited simultaneously at the opposite ends; the detonation wave propagates along the explosive wires and induces axisymmetrical underwater shock waves which, upon the detonation is completed, interact with each other (see Fig. 1; this problem was also studied experimentally and our CFD results are in good agreement with the experimental data).

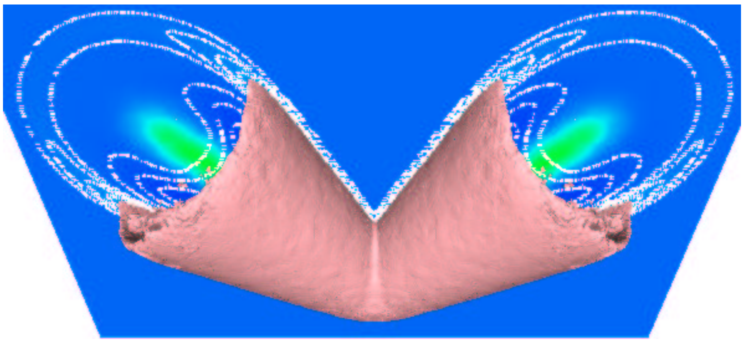

Fig. 1. Two submerged segments of mild detonation fuse are attached to each other at $2 \times 43^\circ$ angle and ignited simultaneously at the opposite ends. The detonation wave propagates along the explosive wires and induces axisymmetrical underwater shock waves (see the isosurface) which, upon the detonation is completed, interact with each other (a regular reflection is observed at the plane of symmetry and that of von Neumann type away from it where the incident angles increases). Pressure contours and color shading according to density values are shown on the plane of symmetry

Another area to which the Artificial Wind schemes can be successively applied is magnetohydrodynamics. A spherically imploding plasma motion in a reversed spherical plasma corona created by a powerful laser is analyzed in [6] as a possible way to generate a strong magnetic field. Another example of magnetohydrodynamic applications is an interesting case of magnetic reconnection (incomplete reconnection and complete reconnection) with sophisticated changes in the magnetic field topology taking place during the evolution of two rotating plasma filaments carrying electric current [7], see Fig. 2.

The Artificial Wind schemes for relativistic hydrodymanics may be developed considering the Lorentz invariance instead of the Galilean one. Although the invariant transformation is quite different, the resulting technique after applying the idea of Artificial Wind is very close to that for the non-relativistic case. The scheme and some results related to the propagation of a relativistic jet through the interstellar medium (Fig. 3) and the relativistic case of the Richtmyer-Meshkov instability are given in [8,9].

It turns out that for magnetohydrodynamics and relativistic hydrodynamics the Artificial Wind schemes result in numerical solutions of quite reasonable quality being at the same time more than one order of magnitude faster than traditional techniques.

4 Conclusion

The application experience demonstrates that our goal has been achieved: the Artificial Wind schemes retain almost the same and simple form for a wide range of hydrodynamical systems while providing numerical solutions at least as accurate as those via traditional upwind schemes at lower (in some cases, considerably lower) computational costs.

Preliminary analysis shows that the Artificial Wind schemes may be also applied for some electrohydrodynamics models describing the motion of electrically charged fluids to integrate both the hydrodynamic and Maxwell equations. Another prospective application area is the simulation of elastic waves in solids and liquids using the elastodynamics equations.

References

1. I.V. Sokolov, E.V. Timofeev, J. Sakai, K. Takayama: 'Artificial Wind – a new framework to construct simple and efficient upwind shock-capturing schemes'. Submitted to J. Comp. Phys. (2001); the PostScript version (33 pages; 1.2Mb) may be requested via timo@ceres.ifs.tohoku.ac.jp
2. E.F. Toro: *"Riemann solvers and numerical methods for fluid dynamics"*, 2nd edn. (Springer-Verlag 1999)
3. A.V. Rodionov: 'Increasing of approximation order of the Godunov scheme'. *Zh. Vychisl. Mat. Mat. Fiz. (Journal of Computational Mathematics and Mathematical Physics)*, **27**(12), 1853 (1987), in Russian
4. T. Saito, P. Voinovich, E. Timofeev, K. Takayama: 'Development and application of high-resolution adaptive numerical techniques in Shock Wave Research Center'. In: *"Godunov methods: Theory and Applicatons"*, Edited Review, ed. by E.F. Toro, (Kluwer Academic/Plenum Publishers 2001), in print
5. E. Timofeev, P. Voinovich, K. Takayama: 'On underwater shock wave propagation induced in the World Ocean by impact of a celestial body'. In: *"SCFS'2000", Proceedings of the Symp. on Computational Fluid Science, Institute of Fluid Science, Tohoku University, Sendai, Japan, August 23, 2000*, ed. by T. Ikohagi, IFS-TM011 (2000), pp. 67–74
6. I.V. Sokolov, J. Sakai: 'Strong magnetic implosion in spherical reversed laser corona: new results and new numerical schemes for MHD simulations'. *Journal of Plasma Fusion Research Series*, **2**, 482 (1998)
7. H.M. Zhang, I.V. Sokolov, J. Sakai: 'Oscillation, shock and fine wave structures arising during the coalescence of two force-free current loops'. *Plasma Phys. Rep.*,**27**(4), 303 (2001)
8. H.M. Zhang, I.V. Sokolov, J. Sakai: 'Towards easy and efficient computational relativistic hydrodynamics'. Submitted to *Proc. Astron. Soc. Japan* (2000)
9. H.M. Zhang, I.V. Sokolov, K. Furusawa, J. Sakai: 'Applications of Artificial Wind numerical scheme for relativistic hydrodynamics in astrophysics'. *Prog. Theor. Phys. Supp.*, **138**, 642 (2000)

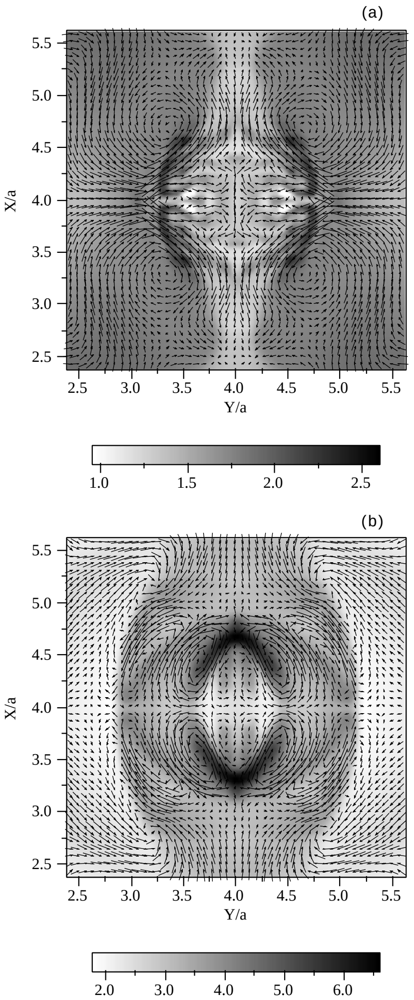

Fig. 2. Magnetic reconnection of two parallel force-free current loops. Fine structure of the plasma flow in the central region: co-helecity **a** and counter-helecity **b** reconnection. Density (gray shading) and velocity vectors are shown.

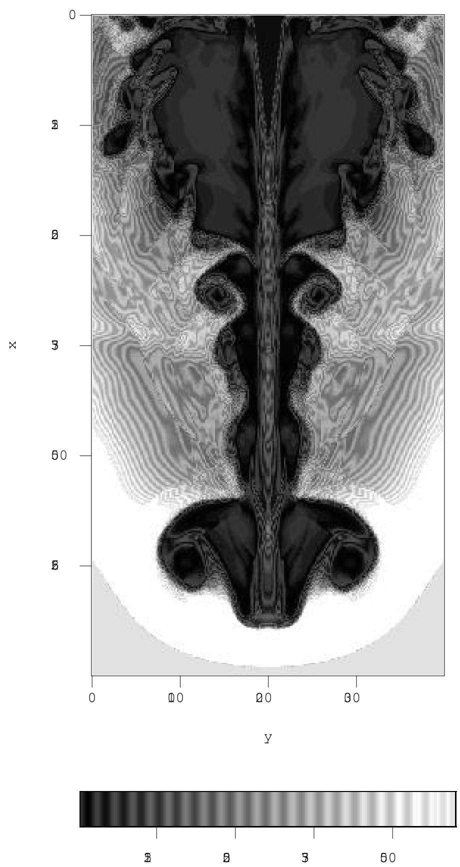

Fig. 3. A relativistic jet in the interstellar medium. Rest-mass density shown in a logarithmic scale with enhanced resolution of low density zones (high density regions are shown in white)