Viscous effects on shock transition over convex and concave walls

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1 Introduction

The reflection of plane shock waves is one of the fundamental topics in shock wave dynamics, and has been investigated by many researchers[1]. It was previously predicted that for shock wave reflection over a rigid wedge whose angle is below the critical angle, a Mach reflection (MR) is theoretically possible and the flow must be self-similar in the absence of viscosity and thermal conductivity. However, if viscous effects are present, the pattern of shock wave reflection is no longer self-similar. The term, viscous effects, used in the present paper generally represents the effects of both viscosity and heat conductivity and is synonymous with Reynolds number effects, since they are inseparable in the macroscopic sense. Recent work revealed that due to viscous effects shock reflection first appears to be a precursory regular reflection, and then after some distrance from the apex of the wedge a delayed transition to MR takes place. This delayed transition to the MR over wedges has been observed experimentally and numerically[5].

On the other hand, for truly unsteady flows, Takayama and Sasaki [9] investigated shock transition over convex and concave walls two decades ago. Their experimental transition angles obtained using models with various radii of curvature, surface roughness and initial angle were scattered for a wide range of shock Mach number. Many analytical models constructed in those days based on the inviscid assumption could predict the pattern of reflection observed in experiment so well that little attention was paid on the role of viscosity. The findings of the delayed transition over wedges stimulate us to reconsider the reason why the data were scattering. We notice that the data scattering observed by Takayama and Sasaki [9] over a circular convex wall with different radii of curvature, initial angle and surface roughness can be more or less explained by taking viscous effects into consideration.

In this work, in order to verify our understanding of viscous effects on the transition angle over convex and concave walls, experiments are conducted and their results are compared for various Reynolds numbers by varying initial pressure. It is found that the reflected shock transition over a convex wall is delayed in low-pressure experiments, and a similar tendency was also observed for experiments over a small-diameter circular wall by Takayama and Sasaki [9]. This indicates that viscous effects are not negligible in the experiment of shock reflection over curved walls. More detailed discussion will be given in the paper.

2 Experiment

A $100mm \times 180mm$ shock tube was used. The shock tube was originally operated by rupturing a diaphragm. In order to generate shock waves at a higher degree of reproducibility, a quick moving piston that initially separated the high-pressure driver gas from the test gas was used a replacement of diaphragms. The scatter of shock wave Mach numbers M_s decreases to $\pm 0.3\%$ for M_s ranging from 1.2 to 5.0 in air [6]. Shock waves and flows are visualized by double exposure holographic interferometry that has been a routine technique for many years in the Shock Wave Research Center (see, for example, [7]). Test gases were argon and nitrogen for experiments over concave and convex surfaces respectively. The incident shock wave Mach number was set to be 2.327 ± 0.007, and initial pressures in the low-pressure section were $1.41kPa$, $7.05kPa$ and $14.1kPa$. All the experiments were conducted at room temperature ($293 \pm 5K$). Numerical values of reflected shock transition criteria for two gases for $M_s = 2.327$ are tabulated below.

Table 1. Shock reflection transition criteria for argon ($\gamma = 5/3$) and nitrogen ($\gamma = 7/5$): θ_e, detachment; θ_s, sonic; θ_N, mechanical equilibrium

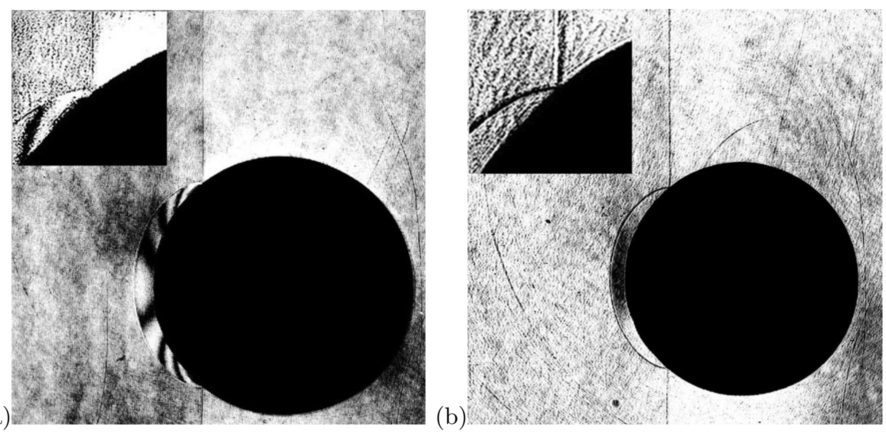
Gas	θ_e	θ_s	θ_N
Ar	53.776	53.924	57.021
N_2	50.757	50.901	57.570

3 Results and discussion

Shock reflections over concave and convex walls have been studied and modeled by Takayama, Ben-Dor, and others [2][3][8][9]. They have investigated thoroughly the effects of wall roughness, curvature of wall surface, and initial angle on the transition angle. Their experimental data indicate that (1) as the surface roughness increases, the wedge angle at which transition takes place decreases for a given Mach number [8] for both convex and concave walls; and (2) as the radius of curvature decreases, the shock transition angle decreases for circular convex walls, while it tends to increase for the concave wall for weak shock waves. We try to interprete how these observations are related to the present experiment from the viewpoint of viscous effects.

A result of shock reflection over a $200mm$ dia. circular cylinder is shown in Fig. 1. The lefthand side photo is taken at initial pressure $p_0 = 7.05kPa$. The local wedge angle, θ_w, which is the intersection angle between the tangent line of the wall surface and the normal of the incident shock front, is about $39.5°$. A short Mach stem is already distinguishable at this moment. It is found that the transition to the MR appears at $\theta_w = 41.9°$. However, it is still a regular reflection in the righthand side photo taken at lower pressure $p_0 = 1.4kPa$ and $\theta_w = 38.2°$. The Mach stem height at different pressures in the present series of experiments is measured and shown as a function of local wedge angle in Fig. 2. It is obvious that low pressure delays the transition from regular reflection to Mach reflection over the convex wall, and hence reduces the transition angle. This trend agrees with the delayed transition over straight wedges.

Suppose that this phenomenon is expressed by the Navier-Stokes equations, a key parameter will be the Reynolds number $Re = \frac{\rho u D}{\mu}$, where μ and u are characteristic viscosity coefficient and velocity respectively. These two parameters are constant at a given shock strength and temperature. The diameter of circular wall surface, D, gives an approprates characteristic length since only circular concave and convex surfaces are considered in this paper. The characteristic density, ρ, is proportional to the initial pressure p_0. The Reynolds number represents an estimate of the relative importance of the non-viscous and viscous forces acting on the gas flow. The smaller Re is, the stronger the viscous effects become. From this dimension analysis it is clear that viscous effects on the transition angle over a large model should be the same as those using a high pressure only if the product of ρD or $p_0 D$ is kept identical. This is why experimental results at low initial pressure and those with small curvature model affect in a similar fashion on the shock transition. Dependence of the transition angle on these conditions observed previously would be an actual evidence of the presence of viscosity. Wall roughness that enhances viscous dissipation in the boundary layer delayed the arrival of signals at shock wave reflection point, and eventually retarded delays the transition.



(a) (b)

Fig. 1. Shock reflection over a circular cylinder under two pressures at $M_s = 2.327$: **a.** $p_0 = 7.05kPa$, $\theta_w = 39.5°$; and **b.** $p_0 = 1.4kPa$, $\theta_w = 38.2°$.

For a circular convex wall, the initial pattern of shock reflection starts with a regular reflection. If viscous effects are neglected, the shape of wall surface that the incident shock has traveled has no influence on the reflection pattern, since signals that propagate in the supersonic region behind the reflection point never catch up with it. Therefore, viscous effects are the dominant factor that may affect the transition angle over the circular convex wall. However, the situation for a concave wall is more complicated. The shock reflection starts with a Mach reflection, and the signals or disturbances initiated from the wall surface that

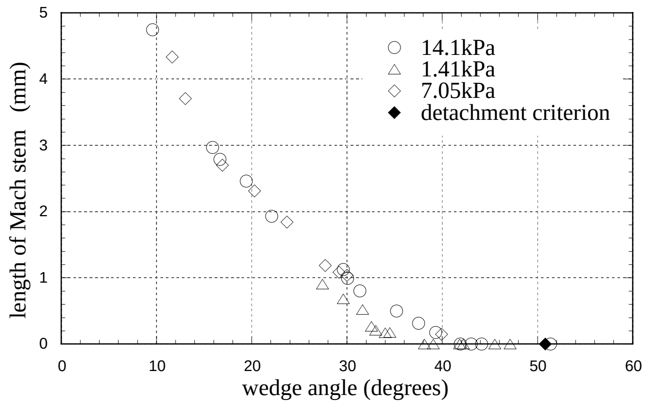


Fig. 2. Mach stem length over a circular cylinder at different pressures

the incident shock has traveled always interact with the shock front. The shape of wall surface, viscous effects, and their mutual interaction, all play a role in the shock transition. This will make the understanding of viscous effects on the transition angle complex.

Four sequential photos of Shock propagation for $M_s = 2.327$ over a $200mm$ dia. concave wall are recorded in Fig. 3. At the early stage of reflection shown in Fig. 3a, the incident shock wave is curved to be normal to the circlarly-shaped surface. Neither the reflected shock wave nor the triple point is visible. This reflection pattern is known as Neumann reflection [4], which is observable in the weak shock reflection over shallow wedges. In Fig. 3a, the local wedge angle is 13.0°. With the propagation of incident wave, the local wedge angle increases, so that their reflection waves get stronger, eventually form a discontinous shock wave. In Fig. 3c, a triple point is clearly visible. This reflection pattern, consisting of three shock confluence is called simple Mach reflection. The triple point moves towards the wall surface, and hits somewhere on the wall. Then the reflection pattern becomes a regular reflection, as shown in the 3d. More details of the entire process were summarized by Ben-Dor and Takayama [2].

Figure 4 shows the results of shock patterns, left column at initial pressure $p_0 = 14.1kPa$ and right column at $p_0 = 1.4kPa$. Enlarged images of shock structure are superimposed on the left upper corner. These interferograms were taken at nearly identical local wedge angles. For the Neumann reflection as shown in Figs. 4a and b, two wave patterns appear to be undistinguishable. Figures 4cd show the shock reflection pattern at angle $\theta_w = 47.6°$. These are are Mach reflection pattern. However, Mach stem height is found to be $5.9mm$ for $p_0 = 14.1kPa$ and 5.1mm for $p_0 = 1.4kPa$. It is clearly found that the Mach stem is shorter at a low initial pressure. Consequently, at a low pressure Mach reflection will terminate earlier, and hence shock reflection transition will take place at small θ_w. Figure 4f shows the reflected shock pattern at a time instant when the Mach reflection terminates at $\theta_w = 70.7°$ and at $p_0 = 1.4kPa$,

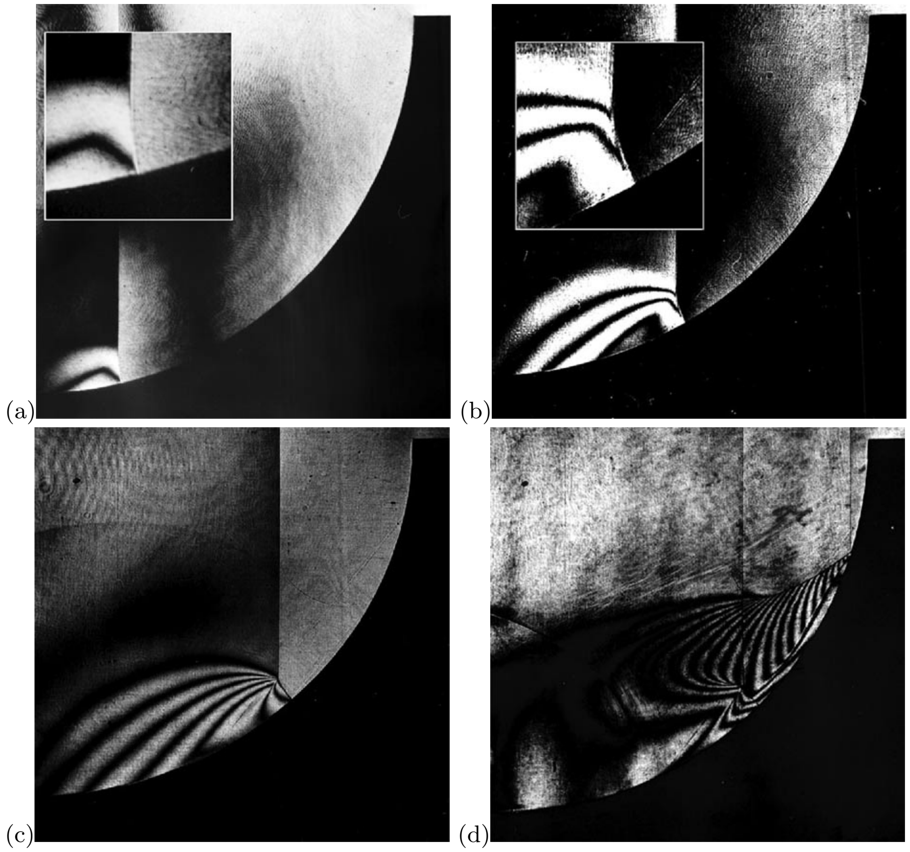


Fig. 3. Shock propagation over a circular concave reflector at M_s =2.327 and $p_0 =$ 14.1 kPa: **a.** $\theta_w = 13.0°$, **b.** $\theta_w = 27.8°$; and **c.** $\theta_w = 40.0°$, **d.** $\theta_w = 72.4°$.

while at $p_0 = 14.1kPa$ the triple point is still at $0.7mm$ above the wall surface. The reflected shock wave pattern for $p_0 = 14.1kPa$ at $\theta_w = 71.8°$ is still Mach reflection. The difference between transition angles at $14.1kPa$ and $1.41kPa$ is about one degree.

The Mach stem height over the concave wedge are measured at various points and plotted in Fig. 5. The laminar Navier-Stokes equations are numerically solved under the same conditions as this experiment, and the numerical results are marked with open circles and open squares. Experimental and numerical result show that the Mach stem height for $p_0 = 1.41$ is below those for $p_0 = 14.1$, although numerical results are slightly below experiment ones. The systematical discrepancy between experiment and simulation is possibly due to finite mesh size in the computation.

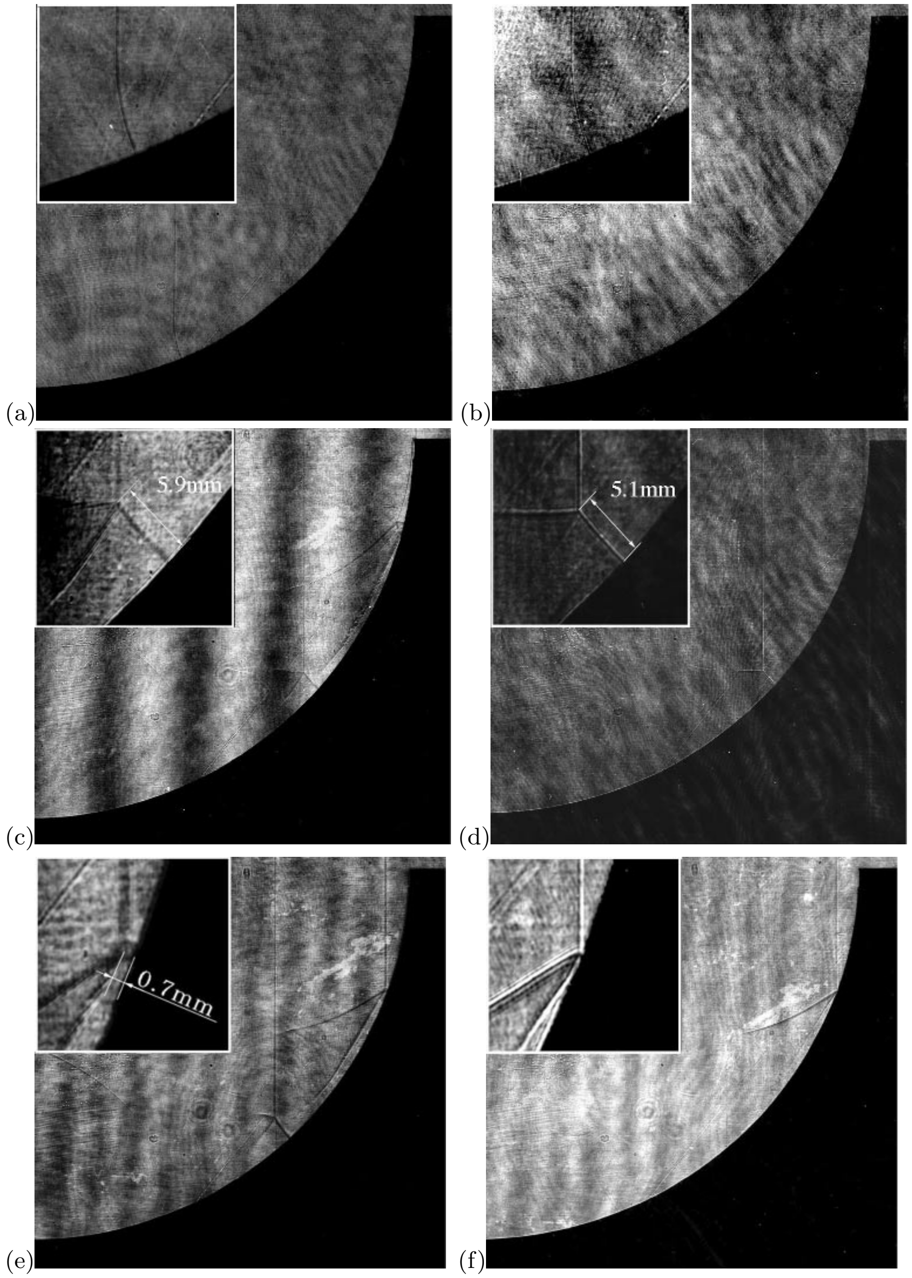


Fig. 4. Shock wave transition over a circular concave reflector under two pressures at $M_s = 2.327$: **a.** $p_0 = 14.1$ kPa, $\theta_w = 22.9°$, **b.** $p_0 = 1.4$ kPa, $\theta_w = 22.8°$; **c.** $p_0 = 14.1$ kPa, $\theta_w = 47.4°$, **d.** $p_0 = 1.4$ kPa, $\theta_w = 47.6°$; **e.** $p_0 = 14.1$ kPa, $\theta_w = 71.0°$, **f.** $p_0 = 1.4$ kPa, $\theta_w = 70.7°$.

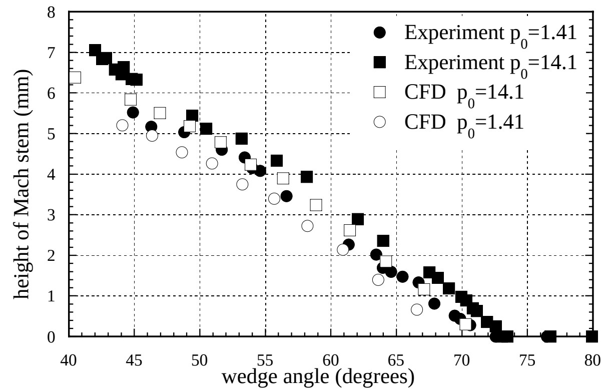


Fig. 5. Mach stem length over a concave wall at two pressures

4 Concluding remarks

Based on the present experiment at various initial pressures and our previous experimental data for models with different radii of curvature, it is clearly found that viscous effects delay the transition over a convex circular wall, and make the transition angles smaller than those obtained over straight wedges. Viscous effects on the transition over a convex surface are more significant and easily detectable than what can be observed over a straight wedge. The delayed transition is observed for large range of shock strength over a circular convex wall, while only for a limited shock strength it is distinguishable for given wedge angles.

The effects of viscosity on the transition over a concave wall are more complex. The present experiment indicates that the low initial pressure promotes the transition from Mach to regular reflection, and decreases its critical transition angle, which shows an opposite tendency that observed in experiments over a small-curvature experiment. The strength of shock wave might affect the role of viscosity in the case of concave wall. More precise experiments are required to clarify the role of viscosity in shock transition over concave walls.

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